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Two-dimensional Brownian random interlacements. (English) Zbl 1453.60136

Potential Anal. 53, No. 2, 727-771 (2020).

Summary: We introduce the model of two-dimensional continuous random interlacements, which is constructed using the Brownian trajectories *conditioned* on not hitting a fixed set (usually, a disk). This model yields the local picture of Wiener sausage on the torus around a late point. As such, it can be seen as a continuous analogue of discrete two-dimensional random interlacements [the authors and *M. Vachkovskaia*, Commun. Math. Phys. 343, No. 1, 129–164 (2016; Zbl 1336.60185)]. At the same time, one can view it as (restricted) Brownian loops through infinity. We establish a number of results analogous to these of the authors [Ann. Probab. 45, No. 6B, 4752–4785 (2017; Zbl 1409.60140)], the authors et al. [loc. cit.], as well as the results specific to the continuous case.

MSC:

60J45 Probabilistic potential theory
60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)
60J65 Brownian motion
60K35 Interacting random processes; statistical mechanics type models; percolation theory

Keywords:

Brownian motion; conditioning; transience; Wiener moustache; logarithmic capacity; Gumbel process

Full Text: DOI

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