Pilipauskaitė, Vytautė; Skorniakov, Viktor; Surgailis, Donatas

Joint temporal and contemporaneous aggregation of random-coefficient AR(1) processes with infinite variance. (English) Zbl 1446.62248

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Summary: We discuss the joint temporal and contemporaneous aggregation of N independent copies of random-coefficient AR(1) processes driven by independent and identically distributed innovations in the domain of normal attraction of an α-stable distribution, $0 < \alpha \leq 2$, as both N and the time scale n tend to infinity, possibly at different rates. Assuming that the tail distribution function of the random autoregressive coefficient regularly varies at the unit root with exponent $\beta > 0$, we show that, for $\beta < \max(\alpha, 1)$, the joint aggregate displays a variety of stable and non-stable limit behaviors with stability index depending on α , β and the mutual increase rate of N and n. The paper extends the results of [the first and third authors, Stochastic Processes Appl. 124, No. 2, 1011–1035 (2014; Zbl 1400.62194)] from $\alpha = 2$ to $0 < \alpha < 2$.

MSC:

62M10 Time series, auto-correlation, regression, etc. in statistics (GARCH)
60F05 Central limit and other weak theorems

Keywords:

autoregressive model; panel data; mixture distribution; infinite variance; long-range dependence; scaling transition; Poisson random measure; asymptotic self-similarity

Software:

longmemo

Full Text: DOI

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