

Ohno, Masahiro

Nef vector bundles on a projective space with first Chern class three. (English) Zbl 1451.14129
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Given a nef vector bundle \mathcal{E} on a projective space \mathbb{P}^n it is well-known that $c_1(\mathcal{E}) \geq 0$. Nef vector bundles \mathcal{E} with $c_1(\mathcal{E}) \leq 2$ were classified by [T. Peternell et al., *Lect. Notes Math.* 1507, 145–156 (1992; [Zbl 0781.14006](#))] analyzing the contraction morphisms of extremal rays. In particular, for $n \geq 2$, $\mathbb{P}(\mathcal{E})$ is a Fano variety. A different proof of the classification was obtained by M. Ohno [“Nef vector bundles on a projective space or a hyperquadric with the first Chern class small”, Preprint, [arXiv:1409.4191](#)] using the twists $\mathcal{E}(d)$.

The paper under review deals with the next case, namely nef vector bundles \mathcal{E} on \mathbb{P}^n (over an algebraically closed field of characteristic zero) with $c_1(\mathcal{E}) = 3$ are completely classified. In particular, one has $0 \leq c_2(\mathcal{E}) \leq c_1(\mathcal{E})^2 = 9$. When $c_2(\mathcal{E}) < 8$, the author proves that the nef vector bundles \mathcal{E} are globally generated. For $c_2 = 8$ and 9, there exist examples of non-globally generated nef vector bundles on the projective plane.

Reviewer: [Joan Pons-Llopis \(Maó\)](#)

MSC:

[14J60](#) Vector bundles on surfaces and higher-dimensional varieties, and their moduli
[14F06](#) Sheaves in algebraic geometry

Keywords:

[nef vector bundles](#); [Fano bundles](#); [spectral sequences](#)

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