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Eventually stable quadratic polynomials over $\mathbb{Q}$. (English) Zbl 1446.37100

New York J. Math. 26, 526-561 (2020).

Let K be a field, $\alpha \in K$, $f \in K[x]$. A pair (f, α) is called eventually stable over K if there exists a constant $C(f;\alpha)$ such that the number of irreducible factors over K of $f^n(x)-\alpha$, where f^n stands for the n-th iterate of f, is at most $C(f;\alpha)$ for all $n \geq 1$. Also, f is eventually stable over K if $(f;0)$ is eventually stable.

The authors prove that the polynomial $f_c(x) = x^2 + 1/c$ is eventually stable over $\mathbb{Q}$ for $c \in \mathbb{Z} \setminus \{0, -1\}$ satisfying $|c| \leq 10^9$, and that $C(f_c, 0) \leq 4$. They also describe many series of c when the n-th iterate of f_c is irreducible over $\mathbb{Q}$ for all $n \geq 1$.

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MSC:

37P15 Dynamical systems over global ground fields
11R09 Polynomials (irreducibility, etc.)
37P05 Arithmetic and non-Archimedean dynamical systems involving polynomial and rational maps
12E05 Polynomials in general fields (irreducibility, etc.)
11R32 Galois theory
11R45 Density theorems

Keywords:

iterated polynomials; irreducible polynomials; rational points; hyperelliptic curves; arboreal Galois representation

Software:

LMFDB; Magma

Full Text: Link

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