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Integer sets containing no arithmetic progressions. (English) Zbl 0721.11007
Acta Math. Hung. 56, No. 1-2, 155-158 (1990).

K. F. Roth [*J. Lond. Math. Soc.* 28, 104-109 (1953; [Zbl 0050.040](#))] showed that a set of positive integers containing no three elements in arithmetic progression has $O(x/\log \log x)$ elements below x . The reviewer improved this to $O(x(\log x)^{-c})$ with an unspecified constant exponent $c > 0$ [*J. Lond. Math. Soc., II. Ser* 35, 385-394 (1987; [Zbl 0589.10062](#))]. The present paper gives the author's account of the problem, the reviewer's version having used ideas from earlier, weaker, unpublished material by the author. The same result is obtained. The discrete circle method is used, but a key lemma of Roth is avoided by a more direct, combinatorial, argument. This results in a shorter, more elegant treatment, in which the exponent c is more readily estimated. The paper states that $c = 1/4$ is admissible, although the necessary calculations are not included. It is clear that $c = 1/3$ is the natural limit of the method.

Reviewer: [D.R.Heath-Brown \(Oxford\)](#)

MSC:

[11B25](#) Arithmetic progressions
[11P55](#) Applications of the Hardy-Littlewood method

Cited in **4** Reviews
Cited in **36** Documents

Keywords:

density; Roth's theorem; no three elements in arithmetic progression; discrete circle method

Full Text: [DOI](#)

References:

- [1] K. F. Roth, On certain sets of integers, *J. London Math. Soc.*, 28 (1953), 104–109. · [Zbl 0050.04002](#) · [doi:10.1112/jlms/s1-28.1.104](#)
- [2] D. R. Heath-Brown, Integer sets containing no arithmetic progressions (to appear). · [Zbl 0589.10062](#)
- [3] J. Pintz, W. L. Steiger and E. Szemerédi, On sets of natural numbers whose difference set contains no squares, to appear in *J. London Math. Soc.* · [Zbl 0651.10031](#)

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