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Partitioning sparse graphs into an independent set and a graph with bounded size components. (English) Zbl 1441.05179

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Summary: We study the problem of partitioning the vertex set of a given graph so that each part induces a graph with components of bounded order; we are also interested in restricting these components to be paths. In particular, we say a graph G admits an $(\mathcal{I},\mathcal{O}_k)$-partition if its vertex set can be partitioned into an independent set and a set that induces a graph with components of order at most k. We prove that every graph G with $\operatorname{mad}(G)<\frac{5}{2}$ admits an $(\mathcal{I},\mathcal{O}_3)$-partition. This implies that every planar graph with girth at least 10 can be partitioned into an independent set and a set that induces a graph whose components are paths of order at most 3. We also prove that every graph G with $\operatorname{mad}(G)<\frac{8k}{3k+1}=\frac{8}{3}\left(1-\frac{1}{3k+1}\right)$ admits an $(\mathcal{I},\mathcal{O}_k)$-partition. This implies that every planar graph with girth at least 9 can be partitioned into an independent set and a set that induces a graph whose components have order at most 9.

MSC:

05C70 Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)
05C42 Density (toughness, etc.)
05C69 Vertex subsets with special properties (dominating sets, independent sets, cliques, etc.)
05C10 Planar graphs; geometric and topological aspects of graph theory

Keywords:

planar graphs; vertex partition; bounded component; discharging method; improper coloring; islands

Full Text: DOI

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