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Solution and asymptotic analysis of a boundary value problem in the spring-mass model of running. (English) Zbl 1434.34052

Nonlinear Dyn. 99, No. 4, 2693-2705 (2020).

Summary: We consider the classic spring-mass model of running which is built upon an inverted elastic pendulum. In a natural way, there arises an interesting boundary value problem for the governing system of two nonlinear ordinary differential equations. It requires us to choose the stiffness to ascertain that after a complete step, the spring returns to its equilibrium position. Motivated by numerical calculations and real data, we conduct a rigorous asymptotic analysis in terms of the Poicaré-Lindstedt series. The perturbation expansion is furnished by an interplay of two time scales what has an significant impact on the order of convergence. Further, we use these asymptotic estimates to prove that there exists a unique solution to the aforementioned boundary value problem and provide an approximation to the sought stiffness. Our results rigorously explain several observations made by other researchers concerning the dependence of stiffness on the initial angle of the stride and its velocity. The theory is illustrated with a number of numerical calculations.

MSC:

34E10 Perturbations, asymptotics of solutions to ordinary differential equations
34B15 Nonlinear boundary value problems for ordinary differential equations
70K60 General perturbation schemes for nonlinear problems in mechanics

Keywords:

singular perturbation theory; boundary value problem; Poincaré-Lindstedt series; elastic pendulum; running

Full Text: DOI

References:

[1] Alasty, A.; Shabani, R., Chaotic motions and fractal basin boundaries in spring-pendulum system, Nonlinear Anal. Real World Appl., 7, 1, 81-95 (2006) · Zbl 1168.70319

[2] Alexander, RM, Optimization and gaits in the locomotion of vertebrates, Physiol. Rev., 69, 4, 1199-1227 (1989)

[3] Alexander, RM, Energy-saving mechanisms in walking and running, J. Exp. Biol., 160, 1, 55-69 (1991)

[4] Alexander, RM, A model of bipedal locomotion on compliant legs, Philos. Trans. R. Soc. Lond. Ser. B Biol. Sci., 338, 1284, 189-198 (1992)

[5] Anderson, T., Biomechanics and running economy, Sports Med., 22, 2, 76-89 (1996)

[6] Behncke, H., A mathematical model for the force and energetics in competitive running, J. Math. Biol., 31, 8, 853-878 (1993) · Zbl 0782.92007

[7] Biewener, A.; Patek, S., Animal Locomotion (2018), Oxford: Oxford University Press, Oxford

[8] Blickhan, R., The spring-mass model for running and hopping, J. Biomech., 22, 11-12, 1217-1227 (1989)

[9] Borelli, G.A.: De motu animalium. Apud Petrum Gosse (1743)

[10] Cavagna, GA; Thys, H.; Zamboni, A., The sources of external work in level walking and running, J. Physiol., 262, 3, 639-657 (1976)

[11] Collins, S.; Ruina, A.; Tedrake, R.; Wisse, M., Efficient bipedal robots based on passive-dynamic walkers, Science, 307, 5712, 1082-1085 (2005)

[12] Cuerno, R.; Ranada, A.; Ruiz-Lorenzo, JJ, Deterministic chaos in the elastic pendulum: a simple laboratory for nonlinear dynamics, Am. J. Phys., 60, 1, 73-79 (1992) · Zbl 1219.70060

[13] Daniels, J.: Daniels' running formula, 3rd edn. Human Kinetics (2013) (ISBN 9781450431835)

[14] Daniels, JT; Yarbrough, R.; Foster, C., Changes in \({\dot{V}}\text{O}_2\) max and running performance with training, Eur. J. Appl. Physiol., 39, 4, 249-254 (1978)

[15] Dickinson, MH; Farley, CT; Full, RJ; Koehl, M.; Kram, R.; Lehman, S., How animals move: an integrative view, Science, 288, 5463, 100-106 (2000)

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[16] Farley, CT; Glasheen, J.; McMahon, TA, Running springs: speed and animal size, J. Exp. Biol., 185, 1, 71-86 (1993)

[17] Ferris, DP; Liang, K.; Farley, CT, Runners adjust leg stiffness for their first step on a new running surface, J. Biomech., 32, 8, 787-794 (1999)

[18] Georgiou, IT, On the global geometric structure of the dynamics of the elastic pendulum, Nonlinear Dyn., 18, 1, 51-68 (1999) · Zbl 0963.70013

[19] Geyer, H.; Seyfarth, A.; Blickhan, R., Spring-mass running: simple approximate solution and application to gait stability, J. Theor. Biol., 232, 3, 315-328 (2005)

[20] Geyer, H.; Seyfarth, A.; Blickhan, R., Compliant leg behaviour explains basic dynamics of walking and running, Proc. R. Soc. B Biol. Sci., 273, 1603, 2861-2867 (2006)

[21] Hastings, S., On the asymptotic growth of solutions to a nonlinear equation, Proc. Am. Math. Soc., 17, 1, 40-47 (1966) · Zbl 0204.10201

[22] Hill, AV, The physiological basis of athletic records, Sci. Mon., 21, 4, 409-428 (1925)

[23] Holm, DD; Lynch, P., Stepwise precession of the resonant swinging spring, SIAM J. Appl. Dyn. Syst., 1, 1, 44-64 (2002) · Zbl 1140.37350

[24] Holmes, MH, Introduction to Perturbation Methods (2012), Berlin: Springer, Berlin

[25] Holmes, P.; Full, RJ; Koditschek, D.; Guckenheimer, J., The dynamics of legged locomotion: models, analyses, and challenges, SIAM Rev., 48, 2, 207-304 (2006) · Zbl 1100.34002

[26] Keller, JB, iA theory of competitive running, Phys. Today, 26, 43 (1973)

[27] Maquet, P., Borelli: De Motu Animalium. A first treatise on biomechanics, Acta Orthop. Belg., 55, 4, 541-546 (1989)

[28] McMahon, TA; Cheng, GC, The mechanics of running: how does stiffness couple with speed?, J. Biomech., 23, 65-78 (1990)

[29] Merker, A.; Kaiser, D.; Hermann, M., Numerical bifurcation analysis of the bipedal spring-mass model, Physica D, 291, 21-30 (2015) · Zbl 1335.37056

[30] Mochon, S.; McMahon, TA, Ballistic walking, J. Biomech., 13, 1, 49-57 (1980)

[31] Nussbaum, MC, Aristotle's De Motu Animalium: Text with Translation, Commentary, and Interpretive Essays (1985), Princeton: Princeton University Press, Princeton

[32] Pavliotis, G.; Stuart, A., Multiscale Methods: Averaging and Homogenization (2008), Berlin: Springer, Berlin · Zbl 1160.35006

[33] Pritchard, WG, Mathematical models of running, SIAM Rev., 35, 3, 359-379 (1993) · Zbl 0782.76109

[34] Rummel, J.; Seyfarth, A., Stable running with segmented legs, Int. J. Robot. Res., 27, 8, 919-934 (2008)

[35] Siegler, S.; Seliktar, R.; Hyman, W., Simulation of human gait with the aid of a simple mechanical model, J. Biomech., 15, 6, 415-425 (1982)

[36] Srinivasan, M.; Holmes, P., How well can spring-mass-like telescoping leg models fit multi-pedal sagittal-plane locomotion data?, J. Theor. Biol., 255, 1, 1-7 (2008) · Zbl 1400.92052

[37] Takahashi, KZ; Worster, K.; Bruening, DA, Energy neutral: the human foot and ankle subsections combine to produce near zero net mechanical work during walking, Sci. Rep., 7, 1, 15404 (2017)

[38] Tibshirani, R.; Albert, J.; Bennett, J.; Cochran, JJ, Who is the fastest man in the world?, Anthology of Statistics in Sports, 311-316 (2005), Philadelphia: SIAM, Philadelphia

[39] Williams, KR; Cavanagh, PR, Relationship between distance running mechanics, running economy, and performance, J. Appl. Physiol., 63, 3, 1236-1245 (1987)

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