

**Chajda, Ivan; Länger, Helmut; Paseka, Jan**

**Algebraic aspects of relatively pseudocomplemented posets.** (English) Zbl 07204021

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Summary: In Chajda and Länger (Math. Bohem. 143, 89-97, 2018) the concept of relative pseudocomplementation was extended to posets. We introduce the concept of a congruence in a relatively pseudocomplemented poset within the framework of Hilbert algebras and we study under which conditions the quotient structure is a relatively pseudocomplemented poset again. This problem is solved e.g. for finite or linearly ordered posets. We characterize relative pseudocomplementation by means of so-called L-identities. We investigate the category of bounded relatively pseudocomplemented posets. Finally, we derive certain quadruples which characterize bounded Hilbert algebras and bounded relatively pseudocomplemented posets up to isomorphism using Glivenko equivalence and implicative semilattice envelope of Hilbert algebras.

**MSC:**

06-XX Order, lattices, ordered algebraic structures

**Keywords:**

relative pseudocomplementation; poset; Hilbert algebra; congruence; convex poset; Dedekind-Macneille completion; Glivenko equivalence; category

**Full Text:** [DOI](#)

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