

**Song, Chao; Jin, Xue; Xu, Chang-Qing**

**Neighbor sum distinguishing total coloring of IC-planar graphs with short cycle restrictions.**

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**Summary:** A graph is IC-planar if it admits a drawing in the plane with at most one crossing per edge, such that two pairs of crossing edges share no common end vertex. For a given graph  $G$ , a proper total coloring  $\phi : V(G) \cup E(G) \rightarrow \{1, 2, \dots, k\}$  is neighbor sum distinguishing if  $f_\phi(u) \neq f_\phi(v)$  for each  $uv \in E(G)$ , where  $f_\phi(v) = \sum_{uv \in E(G)} \phi(uv) + \phi(v)$ ,  $v \in V(G)$ . The smallest integer  $k$  in such a coloring of  $G$  is the neighbor sum distinguishing total chromatic number, denoted by  $\chi''_\Sigma(G)$ . In this paper, by using the discharging method, we prove that  $\chi''_\Sigma(G) \leq \max\{\Delta(G) + 3, 10\}$  if  $G$  is a triangle free IC-planar graph and  $\chi''_\Sigma(G) \leq \max\{\Delta(G) + 3, 13\}$  if  $G$  is an IC-planar graph without adjacent triangles, where  $\Delta(G)$  is the maximum degree of  $G$ .

**MSC:**

**05C15** Coloring of graphs and hypergraphs

**05C10** Planar graphs; geometric and topological aspects of graph theory

**Keywords:**

neighbor sum distinguishing total coloring; IC-planar graph; discharging method

**Full Text:** [DOI](#)

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