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Substitutions and bounded remainder sets. (Russian. English summary) Zbl 1445.11076
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One can begin with the author's abstract:

“The paper is devoted to the multidimensional problem of distribution of fractional parts of a linear function. A subset of a multidimensional torus is called a bounded remainder set if the remainder term of the multidimensional problem of the distribution of the fractional parts of a linear function on this set is bounded by an absolute constant. We are interested not only in the individual bounded remainder sets but also in toric tilings into such sets.

A new class of tilings of a d -dimensional torus into sets of $(d + 1)$ types is introduced. These tilings are defined in combinatorics and geometric terms and are called generalized exchanged tilings. It is proved that all generalized exchanged toric tilings consist of bounded remainder sets. Corresponding estimate of the remainder term is effective. We also find conditions that ensure that the estimate of the remainder term for the sequence of generalized exchanged toric tilings does not depend on the concrete tiling in the sequence.

Using the Arnoux-Ito theory of geometric substitutions we introduce a new class of generalized exchanged tilings of multidimensional tori into bounded remainder sets with an effective estimate of the remainder term. Earlier similar results were obtained in the twodimensional case for one specific substitution. This is a geometric version of well-known Rauzy substitution. With the help of the passage to the limit, another class of generalized exchanged toric tilings into bounded remainder sets with fractal boundaries is constructed (so-called generalized Rauzy fractals).”

A survey on the problem on constructing tilings of multidimensional tori into sets of bounded remainder is given, the interest to such tilings is justified.

This paper consists of the following items:

- Exchanged tilings.
- Geometric substitutions.
- Geometric substitutions and generalized exchanged tilings.
- The Rauzy fractals.

The author notes that the results proven in this paper are generalizations of certain known results. Relations between some results of this research and known results are discussed. Some open problems are described.

Reviewer: [Symon Serbenyuk \(Kyiv\)](#)

MSC:

- [11K06](#) General theory of distribution modulo 1
- [11K38](#) Irregularities of distribution, discrepancy
- [11K55](#) Metric theory of other algorithms and expansions; measure and Hausdorff dimension

Keywords:

[uniform distribution](#); [bounded remainder sets](#); [toric tilings](#); [unimodular Pisot substitutions](#); [geometric substitutions](#)

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