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**Time discretization of an initial value problem for a simultaneous abstract evolution equation applying to parabolic-hyperbolic phase-field systems.** (English) Zbl 1437.65120

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**Summary:** This article deals with a simultaneous abstract evolution equation. This includes a parabolic-hyperbolic phase-field system as an example which consists of a parabolic equation for the relative temperature coupled with a semilinear damped wave equation for the order parameter (see e.g., [*M. Grasselli* and *V. Pata*, Adv. Math. Sci. Appl. 13, No. 2, 443–459 (2003; [Zbl 1057.37068](#)); Commun. Pure Appl. Anal. 3, No. 4, 849–881 (2004; [Zbl 1079.35022](#)); *M. Grasselli* et al., Commun. Pure Appl. Anal. 5, No. 4, 827–838 (2006; [Zbl 1134.35017](#)); *H. Wu* et al., Math. Models Methods Appl. Sci. 17, No. 1, 125–153 (2007; [Zbl 1120.35024](#)); J. Math. Anal. Appl. 329, No. 2, 948–976 (2007; [Zbl 1154.35329](#))]). On the other hand, a time discretization of an initial value problem for an abstract evolution equation has been studied (see e.g., [*P. Colli* and *A. Favini*, Int. J. Math. Math. Sci. 19, No. 3, 481–494 (1996; [Zbl 0859.35077](#))] and [*G. Schimperna*, J. Differ. Equations 164, No. 2, 395–430 (2000; [Zbl 0978.35075](#))] has established existence of solutions to an abstract problem applying to a nonlinear phase-field system of Caginalp type on a bounded domain by employing a time discretization scheme. In this paper we focus on a time discretization of a simultaneous abstract evolution equation applying to parabolic-hyperbolic phase-field systems. Moreover, we can establish an error estimate for the difference between continuous and discrete solutions.

**MSC:**

- [65M12](#) Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
- [65M15](#) Error bounds for initial value and initial-boundary value problems involving PDEs
- [35A35](#) Theoretical approximation in context of PDEs
- [47N20](#) Applications of operator theory to differential and integral equations
- [35G30](#) Boundary value problems for nonlinear higher-order PDEs
- [35L70](#) Second-order nonlinear hyperbolic equations
- [35A01](#) Existence problems for PDEs: global existence, local existence, non-existence

Cited in 1 Document

**Keywords:**

simultaneous abstract evolution equations; parabolic-hyperbolic phase-field systems; existence; time discretizations; error estimates

**Full Text:** [DOI](#)

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