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**Nonlocal elastica model for sparse reconstruction.** (English) Zbl 1455.94040  
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Summary: In view of the exceptional ability of curvature in connecting missing edges and structures, we propose novel sparse reconstruction models via the Euler's elastica energy. In particular, we firstly extend the Euler's elastica regularity into the nonlocal formulation to fully take the advantages of the pattern redundancy and structural similarity in image data. Due to its non-convexity, non-smoothness and nonlinearity, we regard both local and nonlocal elastica functional as the weighted total variation for a good trade-off between the runtime complexity and performance. The splitting techniques and alternating direction method of multipliers (ADMM) are used to achieve efficient algorithms, the convergence of which is also discussed under certain assumptions. The weighting function occurred in our model can be well estimated according to the local approach. Numerical experiments demonstrate that our nonlocal elastica model achieves the state-of-the-art reconstruction results for different sampling patterns and sampling ratios, especially when the sampling rate is extremely low.

**MSC:**

- 94A08 Image processing (compression, reconstruction, etc.) in information and communication theory
- 68U10 Computing methodologies for image processing
- 49M41 PDE constrained optimization (numerical aspects)
- 42C40 Nontrigonometric harmonic analysis involving wavelets and other special systems

**Keywords:**

Euler's elastica; nonlocal regularization; sparse reconstruction; ADMM

**Software:**

DLMRI-Lab; EdgeCS

**Full Text:** [DOI](#)

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