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A formal system of reduction paths for parallel reduction. (English) Zbl 1433.68190

Theor. Comput. Sci. 813, 327-340 (2020).

Summary: We introduce a formal system of reduction paths as a category-like structure induced from a digraph. Our motivation behind this work comes from a quantitative analysis of reduction systems based on the perspective of computational cost and computational orbit. From the perspective, we define a formal system of reduction paths for parallel reduction, wherein reduction paths are generated from a quiver by means of three path-operators. Next, we introduce an equational theory and reduction rules for the reduction paths, and show that the rules on paths are terminating and confluent so that normal paths are obtained. Following the notion of normal paths, a graphical representation of reduction paths is provided. Then we show that the reduction graph is a plane graph, and unique path and universal common-reduct properties are established. Finally, a set of transformation rules from a conversion sequence to a reduction path leading to the universal common-reduct is given under a certain strategy.

MSC:

68Q42 Grammars and rewriting systems
03B40 Combinatory logic and lambda calculus
68R10 Graph theory (including graph drawing) in computer science

Keywords:

Church-Rosser theorem; λ-calculus; parallel reduction; plane graph; quiver; Takahashi translation

Full Text: DOI

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