Kang, Dong Yeap

Sparse highly connected spanning subgraphs in dense directed graphs. (English) Zbl 07186350
Comb. Probab. Comput. 28, No. 3, 423-464 (2019).

Summary: Mader proved that every strongly k-connected n-vertex digraph contains a strongly k-connected spanning subgraph with at most $2kn-2k^2$ edges, where equality holds for the complete bipartite digraph $DK_{k,n-k}$. For dense strongly k-connected digraphs, this upper bound can be significantly improved. More precisely, we prove that every strongly k-connected n-vertex digraph D contains a strongly k-connected spanning subgraph with at most $kn+800k(k+\overline{\Delta}(D))$ edges, where $\overline{\Delta}(D)$ denotes the maximum degree of the complement of the underlying undirected graph of a digraph D. Here, the additional term $800k(k+\overline{\Delta}(D))$ is tight up to multiplicative and additive constants. As a corollary, this implies that every strongly k-connected n-vertex semicomplete digraph contains a strongly k-connected spanning subgraph with at most $kn+800k^2$ edges, which is essentially optimal since $800k^2$ cannot be reduced to the number less than $k(k-1)/2$.

We also prove an analogous result for strongly k-arc-connected directed multigraphs. Both proofs yield polynomial-time algorithms.

MSC:

05C20 Directed graphs (digraphs), tournaments
05C40 Connectivity

Full Text: DOI

References:

[1] Bang-Jensen, J. (2009) Problems and conjectures concerning connectivity, paths, trees and cycles in tournament-like digraphs.Discrete Math.3095655-5667. · Zbl 1207.05067

[2] Bang-Jensen, J. and Gutin, G. (2009) Digraphs: Theory, Algorithms and Applications, second edition, Springer Monographs in Mathematics, Springer. · Zbl 1170.05002

[3] Bang-Jensen, J. and Havet, F. (2018) Tournaments and Semicomplete Digraphs, Springer, pp. 35-124. · Zbl 1407.05102

[4] Bang-Jensen, J., Huang, J. and Yeo, A. (2003) Strongly connected spanning subdigraphs with the minimum number of arcs in quasi-transitive digraphs.SIAM J. Discrete Math.16335-343. · Zbl 1029.05063

[5] Bang-Jensen, J., Huang, J. and Yeo, A. (2004) Spanning k-arc-strong subdigraphs with few arcs in k-arc-strong tournaments.J. Graph Theory46265-284. · Zbl 1057.05039

[6] Bang-Jensen, J. and Yeo, A. (2001) The minimum spanning strong subdigraph problem for extended semicomplete digraphs and semicomplete bipartite digraphs.J. Algorithms411-19. · Zbl 1002.68110

[7] Berg, A. R. and Jordán, T. (2005) Minimally k-edge-connected directed graphs of maximal size.Graphs Combin.2139-50. · Zbl 1062.05079

[8] Cheriyan, J. and Thurimella, R. (2000) Approximating minimum-size k-connected spanning subgraphs via matching.SIAM J. Comput.30528-560. · Zbl 1049.90104

[9] Dalmazzo, M. (1977) Nombre d'arcs dans les graphes k-arc-fortement connexes minimaux. CR Acad. Sci. Paris Sér. A-B285A341-A344. · Zbl 0367.05037

[10] Edmonds, J. (1973) Edge-disjoint branchings. In Combinatorial Algorithms (Rustin, B., ed.), Academic Press, pp. 91-96.

[11] Garey, M. R. and Johnson, D. S. (1979) Computers and Intractability: A Guide to the Theory of NP-Completeness. Freeman. · Zbl 0411.68039

[12] Hopcroft, J. E. and Karp, R. M. (1973) An n^5/2 algorithm for maximum matchings in bipartite graphs.SIAM J. Comput.2225-231. · Zbl 0266.05114

[13] Kang, D. Y., Kim, J., Kim, Y. and Suh, G. (2017) Sparse spanning k-connected subgraphs in tournaments.SIAM J. Discrete Math.312206-2227. · Zbl 1371.05104

[14] Kim, J., Kühn, D. and Osthus, D. (2016) Bipartitions of highly connected tournaments.SIAM J. Discrete Math.30895-911. · Zbl 1336.05050

[15] Kühn, D., Lapinskas, J., Osthus, D. and Patel, V. (2014) Proof of a conjecture of Thomassen on Hamilton cycles in highly connected tournaments. Proc. Lond. Math. Soc.(3)109733-762. · Zbl 1302.05069

[16] Kühn, D., Osthus, D. and Townsend, T. (2016) Proof of a tournament partition conjecture and an application to 1-factors

with prescribed cycle lengths.Combinatorica36451-469. · Zbl 1389.05058

[17] Mader, W. (1985) Minimal n-fach zusammenhängende Digraphen.J. Combin. Theory Ser. B38102-117. · Zbl 0578.05045

[18] Mader, W. (1996) On vertices of degree n in minimally n-connected graphs and digraphs. In Combinatorics: Paul Erdős is Eighty, (Miklós, D.et al., eds), Vol. 2, János Bolyai Mathematical Society, pp. 423-449. · Zbl 0849.05048

[19] Pokrovskiy, A. (2015) Highly linked tournaments.J. Combin. Theory Ser. B115339-347. · Zbl 1319.05063

[20] Pokrovskiy, A. (2017) Edge disjoint Hamiltonian cycles in highly connected tournaments.Int. Math. Res. Not.2017429-467. · Zbl 1405.05097

[21] Thomassen, C. (1982) Edge-disjoint Hamiltonian paths and cycles in tournaments. Proc. London Math. Soc.(3)45151-168. · Zbl 0486.05049

[22] Vetta, A. (2001) Approximating the minimum strongly connected subgraph via a matching lower bound. In Proceedings of the Twelfth Annual ACM-SIAM Symposium on Discrete Algorithms, SIAM, pp. 417-426. · Zbl 0987.05090