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On a Calabi-type estimate for pluriclosed flow. (English) Zbl 1436.53074

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Summary: The regularity theory for pluriclosed flow hinges on obtaining C^α regularity for the metric assuming uniform equivalence to a background metric. This estimate was established in [14] by an adaptation of ideas from Evans-Krylov, the key input being a sharp differential inequality satisfied by the associated 'generalized metric' defined on $T \oplus T^*$. In this work we give a sharpened form of this estimate with a simplified proof. To begin we show that the generalized metric itself evolves by a natural curvature quantity, which leads quickly to an estimate on the associated Chern connections analogous to, and generalizing, Calabi-Yau's C^3 estimate for the complex Monge-Ampère equation.

MSC:

53E30 Flows related to complex manifolds (e.g., Kähler-Ricci flows, Chern-Ricci flows)
35K55 Nonlinear parabolic equations

Keywords:

complex geometry; non-Kähler; pluriclosed flow; generalized complex geometry

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