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**On irregularities of Fourier transforms of regular holonomic  $\mathcal{D}$ -modules.** (English)

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Summary: We study Fourier transforms of regular holonomic  $\mathcal{D}$ -modules. By using the theory of Fourier-Sato transforms of enhanced ind-sheaves developed by Kashiwara-Schapira and D'Agnolo-Kashiwara, a formula for their enhanced solution complexes will be obtained. Moreover we show that some parts of their characteristic cycles and irregularities are expressed by the geometries of the original  $\mathcal{D}$ -modules.

**MSC:**

**32C38** Sheaves of differential operators and their modules,  $D$ -modules

**32S60** Stratifications; constructible sheaves; intersection cohomology (complex-analytic aspects)

**34M35** Singularities, monodromy and local behavior of solutions to ordinary differential equations in the complex domain, normal forms

**35A27** Microlocal methods and methods of sheaf theory and homological algebra applied to PDEs

**Keywords:**

$\mathcal{D}$ -modules; Fourier transforms; irregular singularities

**Full Text:** [DOI](#)

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