AlSharawi, Ziyad; **Pal, Saheb**; **Pal, Nikhil**; **Chattopadhyay, Joydev**

A discrete-time model with non-monotonic functional response and strong Allee effect in prey. (English) Zbl 1435.92049

J. Difference Equ. Appl. 26, No. 3, 404-431 (2020).

Summary: In this paper, we investigate the impact of strong Allee effect on the stability of a discrete-time predator-prey model with a non-monotonic functional response. The dynamics of discrete-time predator-prey models with strong Allee effect is studied earlier. But, the mathematical investigations of predator-prey dynamics in discrete-time set up with Holling type-IV functional response and strong Allee effect in prey are lacking. The proposed model supports the coexistence of two steady states, and the mathematical features of the model are analyzed based on local stability and bifurcation theory. By considering the Allee parameter as the bifurcation parameter, we provide sufficient conditions for the flip and the Neimark-Sacker bifurcations. We observe that Allee parameter plays a significant role in the dynamics of the system.

MSC:

92D25 Population dynamics (general)
34C23 Bifurcation theory for ordinary differential equations

Keywords:

predator-prey model; Allee effect; local stability; flip bifurcation; Neimark-Sacker bifurcation; bistability

Full Text: DOI

References:

[1] Al-Kahby, H., Dannan, F., and Elaydi, S., Non-standard discretization methods for some biological models, in Applications of Nonstandard Finite Difference Schemes (Atlanta, GA, 1999), World Scientific, River Edge, 2000, pp. 155-180. · Zbl 0989.65143

[2] Allee, W. C., Animal Aggregations: A Study in General Sociology (1931), University of Chicago Press: University of Chicago Press, Chicago, IL

[3] Celik, C.; Duman, O., Allee effect in a discrete-time predator-prey system, Chaos Soliton. Fract., 40, 1956-1962 (2009) · Zbl 1198.34084

[4] Cheng, L.; Cao, H., Bifurcation analysis of a discrete-time ratio-dependent predator-prey model with Allee effect, Commun. Nonlinear. Sci. Numer. Simulat., 38, 288-302 (2016)

[5] Collings, J. B., The effects of the functional response on the bifurcation behavior of a mite predator-prey interaction model, J. Math. Biol., 36, 2, 149-168 (1997) · Zbl 0890.92021

[6] Courchamp, F.; Berec, L.; Gascoigne, J., Allee Effects in Ecology and Conservation (2008), Oxford University Press: Oxford University Press, Oxford

[7] Courchamp, F.; Clutton-Brock, T.; Grenfell, B., Inverse density dependence and the Allee effect, Trends. Ecol. Evol. (Amst.), 14, 10, 405-410 (1999)

[8] Courchamp, F.; Grenfell, B. T.; Clutton-Brock, T. H., Impact of natural enemies on obligately cooperative breeders, Oikos, 91, 311-322 (2000)

[9] Dai, L., Nonlinear Dynamics of Piecewise Constant Systems and Implementation of Piecewise Constant Arguments (2008), World Scientific: World Scientific, Singapore · Zbl 1154.70001

[10] Elaydi, S. N.; Sacker, R. J., Population models with Allee effect: A new model, J. Biol. Dyn., 4, 4, 397-408 (2010) · Zbl 1342.92166

[11] Ferdy, J. B.; Austerlitz, F.; Moret, J.; Gouyon, P. H.; Godelle, B., Pollinator-induced density dependence in deceptive species, Oikos, 87, 3, 549-560 (1999)

[12] Freedman, H. I.; Wolkowicz, G. S.K., Predator-prey systems with group defence: The paradox of enrichment revisited, Bull. Math. Biol., 48, 5-6, 493-508 (1986) · Zbl 0612.92017

[13] Groom, M. J., Allee effects limit population viability of an annual plant, Am. Nat., 151, 6, 487-496 (1998)

[14] Guckenheimer, J.; Holmes, P. J., Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields (2013), Springer Science \& Business Media: Springer Science \& Business Media, New York, NY

[15] Hastings, A.; Powell, T., Chaos in three-species food chain, Ecology, 72, 896-903 (1991)

[16] Holmes, J.; Bethel, W., Modification of intermediate host behaviour by parasites, Zool. J. Linn. Soc., 51, Suppl. 1, 123-149 (1972)

[17] Hu, Z.; Teng, Z.; Zhang, L., Stability and bifurcation analysis of a discrete predator-prey model with non-monotonic functional response, Nonlinear Anal. Real World Appl., 12, 2356-2377 (2011) · Zbl 1215.92063

[18] Kapçak, S.; Elaydi, S.; Ufuktepe, Ü., Stability of a predator-prey model with refuge effect, J. Differ. Equ. Appl., 22, 7, 989-1004 (2016)

[19] Kuznetsov, Y. A., Elements of Applied Bifurcation Theory, Vol. 112 (2004), Springer-Verlag: Springer-Verlag, New York, NY · Zbl 1082.37002

[20] Kuussaari, M.; Saccheri, I.; Camara, M.; Hanski, I., Allee effect and population dynamics in the glanville fritillary butterfly, Oikos, 82, 384-392 (1998)

[21] Lotka, A. J., Elements of Physical Biology (1925), Williams and Wilkins: Williams and Wilkins, Baltimore, MD · Zbl 51.0416.06

[22] May, R. M., Biological populations with nonoverlapping generations: stable points, stable cycles, and chaos, Science, 186, 4164, 645-647 (1974)

[23] May, R. M., Simple mathematical models with very complicated dynamics, Nature, 261, 459-467 (1976) · Zbl 1369.37088

[24] Pal, S.; Pal, N.; Chattopadhyay, J., Hunting cooperation in a discrete-time predator-prey system, Int. J. Bifurcat. Chaos, 28, 7 (2018) · Zbl 1392.37020

[25] Pal, S.; Sasmal, S. K.; Pal, N., Chaos control in a discrete-time predator-prey model with weak Allee effect, Int. J. Biomath., 11, 7 (2018) · Zbl 1401.37096

[26] Rana, S.; Bhowmick, A. R.; Bhattacharya, S., Impact of prey refuge on a discrete time predator-prey system with Allee effect, Int. J. Bifurcat. Chaos, 24 (2014) · Zbl 1301.39013

[27] Schreiber, S. J., Allee effects, extinctions, and chaotic transients in simple population models, Theor. Popul. Biol., 64, 2, 201-209 (2003) · Zbl 1104.92053

[28] Sokol, W.; Howell, J. A., Kinetics of phenol oxidation by washed cells, Biotechnol. Bioeng., 23, 9, 2039-2049 (1981)

[29] Stephens, P. A.; Sutherland, W. J., Consequences of the Allee effect for behaviour, ecology and conservation, Trends. Ecol. Evol. (Amst.), 14, 10, 401-405 (1999)

[30] Stoner, A. W.; Ray-Culp, M., Evidence for Allee effects in an over-harvested marine gastropod: Density-dependent mating and egg production, Mar. Ecol. Prog. Ser., 202, 297-302 (2000)

[31] Tener, J. S., Muskoxen in Canada: A Biological and Taxonomic Review, Vol. 2 (1965), Dept. of Northern Affairs and National Resources, Canadian Wildlife Service: Dept. of Northern Affairs and National Resources, Canadian Wildlife Service, Ottawa

[32] Volterra, V., Variazioni e fluttuaziono del numero di individui in specie animali conviventi, Memorie Della Reale Accademia Nazionale Dei Lincei, 2, 31-113 (1926) · Zbl 52.0450.06

[33] Wang, C.; Li, X., Further investigations into the stability and bifurcation of a discrete predator-prey model, J. Math. Anal. Appl., 422, 920-939 (2015) · Zbl 1310.37045

[34] Wang, W. X.; Zhang, Y. B.; Liu, C. Z., Analysis of a discrete-time predator-prey system with Allee effect, Ecol. Complex., 8, 1, 81-85 (2011)

[35] Wolkowicz, G. S.K., Bifurcation analysis of a predator-prey system involving group defence, SIAM. J. Appl. Math., 48, 3, 592-606 (1988) · Zbl 0657.92015

[36] Wu, D.; Zhao, H., Complex dynamics of a discrete predator-prey model with the prey subject to the Allee effect, J. Differ. Equ. Appl., 23, 11, 1765-1806 (2017) · Zbl 1382.92230

[37] Xiao, D.; Ruan, S., Global analysis in a predator-prey system with nonmonotonic functional response, SIAM. J. Appl. Math., 61, 4, 1445-1472 (2001) · Zbl 0986.34045

[38] Zhang, L.; Zou, L., Bifurcations and control in a discrete predator-prey model with strong Allee effect, Int. J. Bifurcat. Chaos, 28, 5 (2018) · Zbl 1390.39060