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Efficient implementation of ADER discontinuous Galerkin schemes for a scalable hyperbolic PDE engine. (English) Zbl 1434.65179
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Summary: In this paper we discuss a new and very efficient implementation of high order accurate arbitrary high order schemes using derivatives discontinuous Galerkin (ADER-DG) finite element schemes on modern massively parallel supercomputers. The numerical methods apply to a very broad class of nonlinear systems of hyperbolic partial differential equations. ADER-DG schemes are by construction communication-avoiding and cache-blocking, and are furthermore very well-suited for vectorization, and so they appear to be a good candidate for the future generation of exascale supercomputers. We introduce the numerical algorithm and show some applications to a set of hyperbolic equations with increasing levels of complexity, ranging from the compressible Euler equations over the equations of linear elasticity and the unified Godunov-Peshkov-Romenski (GPR) model of continuum mechanics to general relativistic magnetohydrodynamics (GRMHD) and the Einstein field equations of general relativity. We present strong scaling results of the new ADER-DG schemes up to 180,000 CPU cores. To our knowledge, these are the largest runs ever carried out with high order ADER-DG schemes for nonlinear hyperbolic PDE systems. We also provide a detailed performance comparison with traditional Runge-Kutta DG schemes.

MSC:

- 65M60** Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs Cited in **6** Documents
- 65M06** Finite difference methods for initial value and initial-boundary value problems involving PDEs
- 35L65** Hyperbolic conservation laws
- 76M12** Finite volume methods applied to problems in fluid mechanics

Keywords:

hyperbolic partial differential equations; high-order discontinuous Galerkin finite element schemes; shock waves and discontinuities; vectorization and parallelization; high performance computing

Software:

ADER-DG; AMRCLAW; CLAWPACK; ECHO; LIBXSMM; NIRVANA; Peano; RAMSES

Full Text: [DOI](#)

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