Beals, Robert; **Gallian, Joseph A.**; **Headley, Patrick**; **Jungreis, Douglas**
Harmonious groups. (English) | Zbl 0718.20013 |

A finite group G is called harmonious if the elements of G can be listed $g_1, g_2, ..., g_n$ so that $G = \{g_1 g_2, g_2 g_3, ..., g_{n-1} g_n, g_n g_1\}$. The main result of this paper is the following theorem: If G is a finite, non-trivial Abelian group, then G is harmonious if and only if G has a non-cyclic or trivial Sylow 2-subgroup and G is not an elementary 2-group (Theorem 6.6). In section 4 of the paper, it is shown that if finite groups G and H are harmonious and H has odd order, then $G \times H$ is harmonious (Theorem 4.1). This result completes the characterization of elegant cycles begun by *G. J. Chang, D. F. Hsu* and *D. G. Rogers* [Congr. Numerantium 32, 181-197 (1981; Zbl 0496.05053)]. In the final section of the paper, the authors also define and investigate harmonious-matched groups.

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MSC:

20D60	Arithmetic and combinatorial problems involving abstract finite groups
20K01	Finite abelian groups
05C25	Graphs and abstract algebra (groups, rings, fields, etc.)
20F65	Geometric group theory

Cited in **1** Review
Cited in **10** Documents

Keywords:

complete mapping; sequencing; harmonious group; Abelian group; finite groups; odd order; harmonious-matched groups

Full Text: DOI

References:

[1] Bruck, R.H, Finite nets, I, numerical invariants, Canad. J. math., 3, 94-107, (1951) · Zbl 0042.38802

[2] Chang, G.J; Hsu, D.F; Rogers, D.G, Additive variations on a graceful theme: some results on harmonious and other related graphs, (), 181-197 · Zbl 0496.05053

[3] Friedlander, R.J; Gordon, B; Miller, M.D, On a group sequencing problem of ringel, (), 307-321

[4] Gorenstein, D, Finite groups, (1968), Harper and Row New York · Zbl 0185.05701

[5] Graham, R.L; Sloane, N.J.A, On additive bases and harmonious graphs, SIAM J. algebraic discrete methods, 4, 382-404, (1980) · Zbl 0499.05049

[6] Gumm, H.P, Encoding of numbers to detect typing errors, Int. J. appl. engng. ed., 2, 61-65, (1986)

[7] Hall, M; Paige, L.J, Complete mappings of finite groups, Pacific J. math., 5, 541-549, (1955) · Zbl 0066.27703

[8] Mann, H.B, The construction of orthogonal Latin fields, Ann. math. statist., 13, 418-423, (1942) · Zbl 0060.02706

[9] Paige, L.J, Neofields, Duke math. J., 16, 39-60, (1949) · Zbl 0040.30501

[10] Ringel, G, Cyclic arrangements of the elements of a group, Notices amer. math. soc., 21, A95-A96, (1974)