**Hayati, Bahman**

**Completely continuous Banach algebras.** (English) Zbl 1449.46038

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Summary: For a Banach algebra $\mathfrak{A}$, we introduce $c.c(\mathfrak{A})$, the set of all $\phi \in \mathfrak{A}^*$ such that $\theta_\phi : \mathfrak{A} \to \mathfrak{A}^*$ is a completely continuous operator, where $\theta_\phi$ is defined by $\theta_\phi(a) = a \cdot \phi$ for all $a \in \mathfrak{A}$. We call $\mathfrak{A}$, a completely continuous Banach algebra if $c.c(\mathfrak{A}) = \mathfrak{A}^*$. We give some examples of completely continuous Banach algebras and a sufficient condition for an open problem raised for the first time by *J. E. Galé* et al. [Trans. Am. Math. Soc. 331, No. 2, 815–824 (1992; Zbl 0761.46037)]: does there exist an infinite dimensional amenable Banach algebra whose underlying Banach space is reflexive? We prove that a reflexive, amenable, completely continuous Banach algebra with the approximation property is trivial.

**MSC:**

46H20 Structure, classification of topological algebras
46H25 Normed modules and Banach modules, topological modules (if not placed in 13-XX or 16-XX)
46B10 Duality and reflexivity in normed linear and Banach spaces

**Keywords:**

amenability; complete continuity; Banach algebra

**Full Text:** DOI

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