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Eigenvalue crossings in Floquet topological systems. (English) Zbl 1434.35138

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Summary: The topology of electrons on a lattice subject to a periodic driving is captured by the three-dimensional winding number of the propagator that describes time evolution within a cycle. This index captures the homotopy class of such a unitary map. In this paper, we provide an interpretation of this winding number in terms of local data associated with the eigenvalue crossings of such a map over a three-dimensional manifold, based on an idea from *F. Nathan* and *M. S. Rudner* ["Topological singularities and the general classification of Floquet-Bloch systems", New J. Phys. 17, No. 12, Article ID 125014, 23 p. (2015; doi:10.1088/1367-2630/17/12/125014)]. We show that, up to homotopy, the crossings are a finite set of points and non-degenerate. Each crossing carries a local Chern number, and the sum of these local indices coincides with the winding number. We then extend this result to fully degenerate crossings and extended submanifolds to connect with models from the physics literature. We finally classify up to homotopy the Floquet unitary maps, defined on manifolds with boundary, using the previous local indices. The results rely on a filtration of the special unitary group as well as the local data of the basic gerbe over it.

MSC:

35Q41 Time-dependent Schrödinger equations and Dirac equations
55M25 Degree, winding number
81Q70 Differential geometric methods, including holonomy, Berry and Hannay phases, Aharonov-Bohm effect, etc. in quantum theory
82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
35P10 Completeness of eigenfunctions and eigenfunction expansions in context of PDEs

Keywords:

Floquet topological insulators; 3d winding number; eigenvalue crossings; local Chern numbers

Full Text: DOI

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