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Descendant log Gromov-Witten invariants for toric varieties and tropical curves. (English)

Zbl 1442.14166

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The authors study genus zero, as well as higher genus Gromov-Witten invariants (in non-superabundant situations) of smooth toric varieties with Psi-class conditions. The main result shows that the tropical description of such invariants coincides with the classical one. The technique that is used to show this correspondence builds on the approach of *T. Nishinou* and *B. Siebert* [Duke Math. J. 135, No. 1, 1–51 (2006; Zbl 1105.14073)]. In particular, it uses logarithmic Gromov-Witten theory and toric degenerations. The authors also allow incidence conditions in the toric boundary for applications to non-toric situations. Moreover, tropically they study arbitrary tropical cycles as incidence conditions, not just affine linear ones as in Nishinou-Siebert.

Reviewer: [Hulya Arguz \(London\)](#)

MSC:

- [14M25](#) Toric varieties, Newton polyhedra, Okounkov bodies
- [14N10](#) Enumerative problems (combinatorial problems) in algebraic geometry
- [14N35](#) Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebraic-geometric aspects)
- [14T15](#) Combinatorial aspects of tropical varieties

Cited in 4 Documents

Keywords:

[tropical Gromov-Witten; Psi class insertion](#)

Software:

[ellipticcovers.lib](#)

Full Text: [DOI](#)

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