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**Nef vector bundles on a quadric surface with the first Chern class (2, 1).** (English)

Zbl 1433.14041

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In the paper under review, the author pursues the classification of rank  $r$  nef vector bundles  $\mathcal{E}$  on a smooth quadric surface  $Q_2 \subset \mathbb{P}^3$  over an algebraically closed field of characteristic zero in terms of the value of the determinant  $\bigwedge^r \mathcal{E} \cong \mathcal{O}_{Q_2}(a, b)$  of  $\mathcal{E}$ . It is well-known that for such a nef bundle  $\mathcal{E}$  it holds that  $a, b \geq 0$  and that  $a = 0$  or  $b = 0$  implies that  $\mathcal{E}$  splits as a direct sum of line bundles. The first non-trivial case, namely  $a = b = 1$ , was classified by *T. Peternell* et al. [Lect. Notes Math. 1507, 145–156 (1992; Zbl 0781.14006)] using strongly the fact that in this case  $\mathbb{P}(\mathcal{E})$  is a Fano manifold.

In this paper the author gives a complete classification of nef bundles for the next open case  $a = 2$  and  $b = 1$  where  $\mathbb{P}(\mathcal{E})$  is no longer a Fano manifold. As a by-product, he proves that all of them are indeed globally generated. The main tool used in this classification is *A. I. Bondal's* results [Math. USSR, Izv. 34, No. 1, 23–42 (1990; Zbl 0692.18002); translation from Izv. Akad. Nauk SSSR, Ser. Mat. 53, No. 1, 25–44 (1989)] on the structure of the derived category of coherent sheaves on  $Q_2$ .

Reviewer: **Joan Pons-Llopis (Maó)**

**MSC:**

- 14J60 Vector bundles on surfaces and higher-dimensional varieties, and their moduli
- 14N30 Adjunction problems
- 14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry

**Keywords:**

nef vector bundles; quadric surfaces; full strong exceptional sequences

**Full Text:** [DOI](#)

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