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Feedback arc number and feedback vertex number of Cartesian product of directed cycles.

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Summary: For a digraph D, the feedback vertex number $\tau(D)$, (resp. the feedback arc number $\tau'(D)$) is the minimum number of vertices, (resp. arcs) whose removal leaves the resultant digraph free of directed cycles. In this note, we determine $\tau(D)$ and $\tau'(D)$ for the Cartesian product of directed cycles $D = \overrightarrow{C_{n_1}}\Box\overrightarrow{C_{n_2}}\Box\cdots\overrightarrow{C_{n_k}}$. Actually, it is shown that $\tau'(D) = n_1n_2\cdots n_k\sum_{i=1}^{k}1/n_i$, and if $n_k \geq \cdots \geq n_1 \geq 3$ then $\tau(D) = n_2\ldots n_k$.

MSC:

05C69 Vertex subsets with special properties (dominating sets, independent sets, cliques, etc.)
05C20 Directed graphs (digraphs), tournaments
05C70 Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)
05C76 Graph operations (line graphs, products, etc.)

Full Text: DOI

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