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Deep learning the hyperbolic volume of a knot. (English) Zbl 1430.57001

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Summary: An important conjecture in knot theory relates the large-N, double scaling limit of the colored Jones polynomial $J_{K,N}(q)$ of a knot K to the hyperbolic volume of the knot complement, $\mathrm{Vol}(K)$. A less studied question is whether $\mathrm{Vol}(K)$ can be recovered directly from the original Jones polynomial ($N = 2$). In this report, we use a deep neural network to approximate $\mathrm{Vol}(K)$ from the Jones polynomial. Our network is robust and correctly predicts the volume with 97.6% accuracy when training on 10% of the data. This points to the existence of a more direct connection between the hyperbolic volume and the Jones polynomial.

MSC:

57-08 Computational methods for problems pertaining to manifolds and cell complexes

57K14 Knot polynomials

68T07 Artificial neural networks and deep learning

Keywords:

machine learning; neural network; topological field theory; knot theory

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Mathematica; SnapPy

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