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Gilbert's disc model with geostatistical marking. (English) Zbl 1431.60010

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Summary: We study a variant of Gilbert's disc model, in which discs are positioned at the points of a Poisson process in $\mathbb{R}^2$ with radii determined by an underlying stationary and ergodic random field $\phi : \mathbb{R}^2 \to [0, \infty)$, independent of the Poisson process. This setting, in which the random field is independent of the point process, is often referred to as geostatistical marking. We examine how typical properties of interest in stochastic geometry and percolation theory, such as coverage probabilities and the existence of long-range connections, differ between Gilbert's model with radii given by some random field and Gilbert's model with radii assigned independently, but with the same marginal distribution. Among our main observations we find that complete coverage of $\mathbb{R}^2$ does not necessarily happen simultaneously, and that the spatial dependence induced by the random field may both increase as well as decrease the critical threshold for percolation.

MSC:

60D05 Geometric probability and stochastic geometry
60K35 Interacting random processes; statistical mechanics type models; percolation theory
60K37 Processes in random environments

Keywords:

continuum percolation; coverage probability; threshold comparison

Full Text: DOI

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