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Neighbor sum distinguishing total colorings of IC-planar graphs with maximum degree 13. (English) Zbl 1434.05057

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Summary: A graph is IC-planar if it admits a drawing on the plane with at most one crossing per edge, such that two pairs of crossing edges share no common end vertex. For a given graph G, a proper total coloring $\phi : V(G) \cup E(G) \to \{1, 2, \dots, k\}$ is called neighbor sum distinguishing if $f_\phi(u) \neq f_\phi(v)$ for each $uv \in E(G)$, where $f_\phi(u)$ is the sum of the color of u and the colors of the edges incident with u. The smallest integer k in such a coloring of G is the neighbor sum distinguishing total chromatic number, denoted by $\chi''_\Sigma(G)$. *M. Pilśniak* and *M. Woźniak* [Graphs Comb. 31, No. 3, 771–782 (2015; Zbl 1312.05054)] conjectured $\chi''_\Sigma(G) \leq \Delta(G) + 3$ for any simple graph with maximum degree $\Delta(G)$. This conjecture was confirmed for IC-planar graph with maximum degree at least 14. In this paper, by using the discharging method, we prove that this conjecture holds for any IC-planar graph G with $\Delta(G) = 13$.

MSC:

05C15 Coloring of graphs and hypergraphs
05C10 Planar graphs; geometric and topological aspects of graph theory

Keywords:

neighbor sum distinguishing total coloring; IC-planar graph; discharging method

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