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Global regularity for the 2D Boussinesq equations with temperature-dependent viscosity.
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Summary: This paper is devoted to the global regularity for the Cauchy problem of the two-dimensional Boussinesq equations with the temperature-dependent viscosity. We prove the global solutions for this system with any positive power of the fractional Laplacian for temperature under the assumption that the viscosity coefficient is sufficiently close to some positive constant. Our obtained result improves considerably the recent results in [*H. Abidi* and *P. Zhang*, *Adv. Math.* 305, 1202–1249 (2017; [Zbl 1353.35220](#))] and [*X. Zhai* et al., *J. Differ. Equations* 267, No. 1, 364–387 (2019; [Zbl 1414.35153](#))]. In addition, a regularity criterion via the velocity is also obtained for this system without the above assumption on the viscosity coefficient.

MSC:

- [35Q35](#) PDEs in connection with fluid mechanics
- [35B65](#) Smoothness and regularity of solutions to PDEs
- [76D03](#) Existence, uniqueness, and regularity theory for incompressible viscous fluids
- [35R11](#) Fractional partial differential equations
- [42B25](#) Maximal functions, Littlewood-Paley theory

Keywords:

Boussinesq equations; variable viscosity; global regularity

Full Text: [DOI](#)

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