

**Al-Ghassani, Asma S.; AlSharawi, Ziyad**

**The effect of maps permutation on the global attractor of a periodic Beverton-Holt model.**

(English) [Zbl 1433.39001](#)

[Appl. Math. Comput.](#) 370, Article ID 124905, 13 p. (2020).

Summary: Consider a  $p$ -periodic difference equation  $x_{n+1} = f_n(x_n)$  with a global attractor. How does a permutation  $[f_{\sigma(p-1)}, \dots, f_{\sigma(1)}, f_{\sigma(0)}]$  of the maps affect the global attractor? In this paper, we limit this general question to the Beverton-Holt model with  $p$ -periodic harvesting. We fix a set of harvesting quotas and give ourselves the liberty to permute them. The total harvesting yield is unchanged by the permutation, but the population geometric-mean may fluctuate. We investigate this notion and characterize the cases in which a permutation of the harvesting quotas has no effect or tangible effect on the population geometric-mean. In particular, as long as persistence is assured, all permutations within the dihedral group give same population geometric-mean. Other permutations may change the population geometric-mean. A characterization theorem has been obtained based on block reflections in the harvesting quotas. Finally, we associate directed graphs to the various permutations, then give the complete characterization when the periodicity of the system is four or five.

**MSC:**

- [39A10](#) Additive difference equations
- [92D25](#) Population dynamics (general)
- [37E15](#) Combinatorial dynamics (types of periodic orbits)
- [37N25](#) Dynamical systems in biology

**Keywords:**

[Beverton-Holt](#); [cycles](#); [permutations](#); [periodic harvesting](#); [combinatorial dynamics](#)

**Full Text:** [DOI](#)

**References:**

- [1] Al-Salman, A.; AlSharawi, Z., A new characterization of periodic oscillations in periodic difference equations, *Chaos Solitons Fract.*, 44, 921-928 (2011) · [Zbl 1271.39012](#)
- [2] Alsedra, L.; Llibre, J.; Misiurewicz, M., *Combinatorial Dynamics and Entropy in Dimension One* (2000), Second Edition, World Scientific · [Zbl 0963.37001](#)
- [3] AlSharawi, Z.; Canovas, J.; Linero, A., Folding and unfolding in periodic difference equations, *J. Math. Anal. Appl.*, 417, 643-659 (2014) · [Zbl 1302.39024](#)
- [4] AlSharawi, Z.; Rhouma, M. B.H., The beverton-holt model with periodic and conditional harvesting, *J. Biol. Dyn.*, 3, 463-478 (2009) · [Zbl 1342.91025](#)
- [5] Art. ID 215875. · [Zbl 1184.92047](#)
- [6] Beverton, R. J.H.; Holt, S. J., *On the Dynamics of Exploited Fish Population* (2004), Blackburn Press: Blackburn Press New Jersey
- [7] Art. ID 5963594. · [Zbl 1370.92126](#)
- [8] Brauer, F.; Sanchez, D. A., Periodic environments and periodic harvesting, *Nat. Resour. Model.*, 16, 233-244 (2003) · [Zbl 1067.92056](#)
- [9] Brauer, F.; Sanchez, D. A., Constant rate population harvesting: equilibrium and stability, *Theoret. Popul. Biol.*, 8, 12-30 (1975) · [Zbl 0313.92012](#)
- [10] Chau, N. P., Destabilising effect of periodic harvest on population dynamics, *Ecol. Modell.*, 127, 1-9 (2000)
- [11] Clark, C. W., *Mathematical Bioeconomics: The Optimal Management of Renewable Resources* (1990), John Wiley & Sons · [Zbl 0712.90018](#)
- [12] Clark, W. G.; Hare, S. R., A conditional constant catch policy for managing the pacific halibut fishery, *North Am. J. Fish. Manag.*, 24, 106-113 (2004)
- [13] Costa, M. I., Harvesting induced fluctuations: insights from a threshold management policy, *Math. Biosci.*, 205, 77-82 (2007) · [Zbl 1106.92069](#)

- [14] Committee on fish stock assessment methods, 1998, (????). Improving Fish Stock Assessments, National Research Council.
- [15] Cushing, J. M.; Henson, S. M., Global dynamics of some periodically forced, monotone difference equations, *J. Differ. Equ. Appl.*, 7, 859-872 (2001) · [Zbl 1002.39003](#)
- [16] Cushing, J. M.; Henson, S. M., A periodically forced Beverton-Holt equation, *J. Differ. Equ. Appl.*, 8, 1119-1120 (2001) · [Zbl 1023.39013](#)
- [17] Elaydi, S.; Sacker, R. J., Population models with allee effect: a new model, *J. Biol. Dyn.*, 4, 397-408 (2010) · [Zbl 1342.92166](#)
- [18] Elaydi, S.; Sacker, R. J., Global stability of periodic orbits of nonautonomous difference equations in population biology and the cushing-henson conjecture, *J. Differ. Equ.*, 208, 258-273 (2005) · [Zbl 1067.39003](#)
- [19] Elaydi, S.; Sacker, R. J., Nonautonomous Beverton-Holt equations and the cushing-henson conjectures, *J. Differ. Equ. Appl.*, 11, 337-346 (2005) · [Zbl 1084.39005](#)
- [20] Franke, J. E.; Yakubu, A., Signature function for predicting resonant and attenuant population 2-cycles, *Bull. Math. Biol.*, 68, 2069-2104 (2006) · [Zbl 1296.92203](#)
- [21] Franke, J. E.; Yakubu, A., Population models with periodic recruitment functions and survival rates, *J. Differ. Equ. Appl.*, 11, 1169-1184 (2005) · [Zbl 1079.92063](#)
- [22] Horn, R. A.; Johnson, C. R., *Matrix Analysis* (1985), Cambridge University Press: Cambridge University Press USA · [Zbl 0576.15001](#)
- [23] Kocic, V. L., A note on the nonautonomous Beverton-Holt model, *J. Differ. Equ. Appl.*, 11, 415-422 (2005) · [Zbl 1084.39007](#)
- [24] Kocic, V. L., A note on the nonautonomous delay Beverton-Holt model, *J. Biol. Dyn.*, 4, 131-139 (2010) · [Zbl 1345.92118](#)
- [25] Kon, R., A note on attenuant cycles of population models with periodic carrying capacities, *J. Differ. Equ. Appl.*, 10, 791-793 (2004) · [Zbl 1056.92046](#)
- [26] Selgrade, J. F.; Roberds, J. H., Global attractors for a discrete selection model with periodic immigration, *J. Differ. Equ. Appl.*, 13, 275-287 (2007) · [Zbl 1110.92032](#)
- [27] Tang, S.; Cheke, R. A.; Xiao, Y., Optimal impulsive harvesting on non-autonomous Beverton-Holt difference equation, *Non-linear Anal.: Theory, Methods Appl.*, 65, 2311-2341 (2006) · [Zbl 1119.39011](#)
- [28] Taussky, O., The role of symmetric matrices in the study of general matrices, *Linear Algebra and Appl.*, 5, 147-154 (1972) · [Zbl 0238.15008](#)
- [29] Turchin, P., *Complex Population Dynamics* (2003), Princeton University Press: Princeton University Press USA · [Zbl 1062.92077](#)
- [30] Xu, C.; Boyce, M. S.; Daley, D. J., Harvesting in seasonal environments, *J. Math. Biol.*, 50, 663-682 (2005) · [Zbl 1066.92057](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.