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On 1-factors with prescribed lengths in tournaments. (English) Zbl 1430.05048

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Summary: We prove that every strongly $10^{50}t$-connected tournament contains all possible 1-factors with at most t components and this is best possible up to constant. In addition, we can ensure that each cycle in the 1-factor contains a prescribed vertex. This answers a question by *D. Kühn* et al. [Combinatorica 36, No. 4, 451–469 (2016; Zbl 1389.05058)].

Indeed, we prove more results on partitioning tournaments. We prove that a strongly $\Omega(k^4tq)$-connected tournament admits a vertex partition into t strongly k-connected tournaments with prescribed sizes such that each tournament contains q prescribed vertices, provided that the prescribed sizes are $\Omega(n)$. This result improves the earlier result of Kühn et al. [loc. cit.]. We also prove that for a strongly $\Omega(t)$-connected n-vertex tournament T and given $2t$ distinct vertices $x_1, \dots, x_t, y_1, \dots, y_t$ of T, we can find t vertex disjoint paths $P_1, \dots, P_t$ such that each path P_i connecting x_i and y_i has the prescribed length, provided that the prescribed lengths are $\Omega(n)$. For both results, the condition of connectivity being linear in t is best possible, and the condition of prescribed sizes being $\Omega(n)$ is also best possible.

MSC:

05C20 Directed graphs (digraphs), tournaments
05C38 Paths and cycles
05C70 Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)

Keywords:

tournament; connectivity; 1-factor; cycle; graph partition

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