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Tan, Ken Seng; **Wei, Pengyu**; **Wei, Wei**; **Zhuang, Sheng Chao**

Optimal dynamic reinsurance policies under a generalized Denneberg's absolute deviation principle. (English) Zbl 1431.91344

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Summary: This paper studies the optimal dynamic reinsurance policy for an insurance company whose surplus is modeled by the diffusion approximation of the classical Cramér-Lundberg model. We assume the reinsurance premium is calculated according to a proposed mean-CVaR premium principle which generalizes Denneberg's absolute deviation principle and expected value principle. Moreover, we require that both ceded loss and retention functions are non-decreasing to rule out moral hazard. Under the objective of minimizing the ruin probability, we obtain the optimal reinsurance policy explicitly and we denote the resulting treaty as the dual excess-of-loss reinsurance. This form of the optimal treaty is new to the literature and lends support to the fact that reinsurance contracts in practice often involve layers. It also demonstrates that reinsurance treaties such as the proportional and the standard excess-of-loss, which are typically found to be optimal in the dynamic reinsurance model, need not be optimal when we consider a more general optimization model. We also consider other generalizations including (i) allowing the insurer to manage its business through both reinsurance and investment; and (ii) N-piecewise mean-CVaR premium principle. In the former case, we not only show that the dual excess-of-loss reinsurance policy remains optimal, but also demonstrate that investing in stock can further enhance insurer's financial stability with lower ruin probability. For the latter case, we establish that the optimal reinsurance treaty can have at most N layers, which is also more consistent with practice.

MSC:

91G05 Actuarial mathematics
91G10 Portfolio theory

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