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Neighbor sum distinguishing total chromatic number of planar graphs without 5-cycles.
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Summary: For a given graph $G = (V(G), E(G))$, a proper total coloring $\varphi : V(G) \cup E(G) \rightarrow \{1, 2, \dots, k\}$ is neighbor sum distinguishing if $f(u) \neq f(v)$ for each edge $uv \in E(G)$, where $f(v) = \sum_{uv \in E(G)} \varphi(uv) + \varphi(v)$, $v \in V(G)$. The smallest integer k in such a coloring of G is the neighbor sum distinguishing total chromatic number, denoted by $\chi''_{\Sigma}(G)$. *M. Piłśniak* and *M. Woźniak* [*Graphs Comb.* 31, No. 3, 771–782 (2015; [Zbl 1312.05054](#))] first introduced this coloring and conjectured that $\chi''_{\Sigma}(G) \leq \Delta(G) + 3$ for any graph with maximum degree $\Delta(G)$. In this paper, by using the discharging method, we prove that for any planar graph G without 5-cycles, $\chi''_{\Sigma}(G) \leq \max\{\Delta(G) + 2, 10\}$. The bound $\Delta(G) + 2$ is sharp. Furthermore, we get the exact value of $\chi''_{\Sigma}(G)$ if $\Delta(G) \geq 9$.

MSC:

05C15 Coloring of graphs and hypergraphs

Keywords:

neighbor sum distinguishing total coloring; discharging method; planar graph

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