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- 92C20 Neural biology
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- 34C15 Nonlinear oscillations and coupled oscillators for ordinary differential equations

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NEURON; Python

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