Climent Vidal, J.; **Cosme Llópez, E.**

A characterization of the n-ary many-sorted closure operators and a many-sorted Tarski irredundant basis theorem. (English) Zbl 07144285

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Summary: A theorem of single-sorted algebra states that, for a closure space (A, J) and a natural number n, the closure operator J on the set A is n-ary if and only if there exists a single-sorted signature Σ and a Σ-algebra $\mathbf{A}$ such that every operation of $\mathbf{A}$ is of an arity $\leq n$ and $J = \mathrm{Sg}\mathbf{A}$, where $\mathrm{Sg}\mathbf{A}$ is the subalgebra generating operator on A determined by $\mathbf{A}$. On the other hand, a theorem of Tarski asserts that if J is an n-ary closure operator on a set A with $n \geq 2$, then, for every $i, j \in \mathrm{IrB}(A, J)$, where $\mathrm{IrB}(A, J)$ is the set of all natural numbers which have the property of being the cardinality of an irredundant basis ($\equiv$ minimal generating set) of A with respect to J , if $i < j$ and $i + 1\{i + 1, \ldots, j - 1\} \cap \mathrm{IrB}(A, J) = \varnothing$, then $j - i \leq n - 1$. In this article we state and prove the many-sorted counterparts of the above theorems. But, we remark, regarding the first one under an additional condition: the uniformity of the many-sorted closure operator.

MSC:

06A15 Galois correspondences, closure operators (in relation to ordered sets)
54A05 Topological spaces and generalizations (closure spaces, etc.)

Keywords:

S-sorted set; delta of Kronecker; support of an S-sorted set; n-ary many-sorted closure operator; uniform many-sorted closure operator; irredundant basis with respect to a many-sorted closure operator

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