Rito, Carlos; **Roulleau, Xavier**; **Sarti, Alessandra**
Explicit Schoen surfaces. (English) Zbl 1428.14071
Algebr. Geom. 6, No. 4, 410-426 (2019).

In this extremely interesting paper, the authors construct in an explicit way the 4-dimensional family of Schoen surfaces by computing equations for their canonical images.

The main result of the paper can be summed up as follows.

Main Result. Let I_4 be the Igusa quartic in $\mathbb{P}^4_{\mathbb{C}}$. There exists a quadric on four parameteres $Q_{a,b,c,d}$ such that for generic values of these parameters the surface

$$X_{40} := I_4 \cap Q_{a,b,c,d}$$

has exactly 40 nodes. The nodes are 2-divisible in the Picard group, and the double cover $S \to X_{40}$ ramified over the nodes is a Schoen surface (i.e., $K_s^2 = 2e(S) = 16$, $q(S) = 4$, $p_g(S) = 5$).

The authors show explicitly that the surface S is not covered by the bidisk $\mathbb{H} \times \mathbb{H}$.

In the remaining part of the paper the authors study a certain particular surface $\overline{S}$ obtained as the double cover of a particular 40-nodal and degree 8 complete intersection surface with a large group of symmetries. Let us denote by $\overline{X_{40}} \subset \mathbb{P}^4_{\mathbb{C}}$ the intersection of the following quadric and quartic:

$$5(x^2 + y^2 + z^2 + w^2 + t^2) - 7(x + y + z + w + t)^2 = 0,$$

$$4(x^4 + y^4 + z^4 + w^4 + t^4 + h^4) - (x^2 + y^2 + z^2 + w^2 + t^2 + h^2)^2 = 0,$$

where $h = -(x + y + z + w + t)$. The surface $\overline{X_{40}}$ has exactly 40 nodes. Let $\overline{S} \to \overline{X_{40}}$ be the double cover branched over the 40 nodes and let $\hat{X}_{40}$ be the minimal resolution of $\overline{X_{40}}$. Using the symmetries (i.e., the permutation group S_5 is a subgroup of the automorphism group of $\overline{X_{40}}$) the authors are able to show that $\overline{S}$ and $\hat{X}_{40}$ have maximal Picard numbers equal to 12 and 52, respectively.

Due to the fact that the review should be rather concise, the reviewer warmly suggest to consult details of the paper with the emphasis of very nice constructions.

Reviewer: Piotr Pokora (Kraków)

MSC:

14J29   Surfaces of general type
14J28 $K3$ surfaces and Enriques surfaces

Keywords:

$K3$ surfaces; irregular surfaces; Lagrangian surfaces; Segre cubic; Igusa quartic

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