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Discussing Hilbert's 24th problem. (English) [Zbl 1441.03006](#)

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This is a presentation of the text of Hilbert's 24th problem, its historical context, the early contributions to it from Hilbert's own entourage, the problems it raises today, together with brief reviews of the contributions to the special issue of the *Philosophical Transactions of the Royal Society* devoted to Hilbert's 24th problem, to which this paper serves as an introduction.

Reviewer: [Victor V. Pambuccian \(Glendale\)](#)

MSC:

- [03-03](#) History of mathematical logic and foundations
- [01A60](#) History of mathematics in the 20th century
- [03F03](#) Proof theory, general (including proof-theoretic semantics)

Keywords:

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