Lacoin, Hubert

Wetting and layering for solid-on-solid. II: Layering transitions, Gibbs states, and regularity of the free energy. (Mouillage et stratification pour le modèle SOS II : transitions de niveau, états de Gibbs et régularité de l'énergie libre.) (English. French summary) Zbl 07128376

J. Éc. Polytech., Math. 7, 1-62 (2020).

Summary: We consider the Solid-on-Solid model interacting with a wall, which is the statistical mechanics model associated with the integer-valued field $(\phi(x))_{x\in\mathbb{Z}^2}$, and the energy functional

$$V(\phi) = \beta \sum_{x\sim y} |\phi(x) - \phi(y)| - \sum_{x} \left(h\mathbf{1}_{\{\phi(x)=0\}} - \infty \mathbf{1}_{\{\phi(x)<0\}} \right).$$

We prove that for β sufficiently large, there exists a decreasing sequence $(h_n^*(\beta))_{n\geq 0}$, satisfying $\lim_{n\to\infty} h_n^*(\beta) = h_w(\beta)$, and such that: (A) The free energy associated with the system is infinitely differentiable on $\mathbb{R} \setminus (\{h_n^*\}_{n\geq 1} \cup h_w(\beta))$, and not differentiable on $\{h_n^*\}_{n\geq 1}$.(B) For each $n \geq 0$ within the interval (h_{n+1}^*, h_n^*) (with the convention $h_0^* = \infty$), there exists a unique translation invariant Gibbs state which is localized around height n, while at a point of non-differentiability, at least two ergodic Gibbs states coexist. The respective typical heights of these two Gibbs states are $n-1$ and n. The value h_n^* corresponds thus to a first order layering transition from level n to level $n-1$. These results combined with those obtained in Part I [the author, Commun. Math. Phys. 362, No. 3, 1007–1048 (2018; Zbl 1398.82023)] provide a complete description of the wetting and layering transition for SOS.

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory
60K37 Processes in random environments
82B27 Critical phenomena in equilibrium statistical mechanics
82B44 Disordered systems (random Ising models, random Schrödinger operators, etc.) in equilibrium statistical mechanics

Keywords:

solid-on-solid; wetting; layering transitions; Gibbs states

Full Text: DOI arXiv

References:

[1] Ahlberg, Daniel; Tassion, Vincent; Teixeira, Augusto, Existence of an unbounded vacant set for subcritical continuum percolation, Electron. Comm. Probab., 23 (2018) · Zbl 1401.60173

[2] Aizenman, Michael, Translation invariance and instability of phase coexistence in the two-dimensional Ising system, Comm. Math. Phys., 73, 1, 83-94 (1980)

[3] Alexander

[4] Alexander; Dunlop, François; Miracle-Solé, Salvador, Layering and wetting transitions for an SOS interface, J. Statist. Phys., 142, 3, 524-576 (2011) · Zbl 1209.82011

[5] Armitstead, K.; Yeomans, A series approach to wetting and layering transitions. II. Solid-on-solid models, J. Phys. A, 21, 1, 159-171 (1988)

[6] Basuev, Ising model in half-space: a series of phase transitions in low magnetic fields., Theoret. and Math. Phys., 153, 2, 1539-1574 (2007) · Zbl 1139.82309

[7] Bissacot, Rodrigo; Fernández, Roberto; Procacci, Aldo, On the convergence of cluster expansions for polymer gases, J. Statist. Phys., 139, 4, 598-617 (2010) · Zbl 1196.82135

[8] Brandenberger, R.; Wayne, Decay of correlations in surface models, J. Statist. Phys., 27, 3, 425-440 (1982)

[9] Bricmont, J.; El Mellouki, A.; Fröhlich, J., Random surfaces in statistical mechanics: roughening, rounding, wetting, ..., J. Statist. Phys., 42, 5-6, 743-798 (1986)

[10] Burton; Cabrera, N.; Frank, The growth of crystals and the equilibrium structure of their surfaces, Philos. Trans. Roy. Soc.

London Ser. A, 243, 299-358 (1951) · Zbl 0043.23402

[11] Caputo, Pietro; Lubetzky, Eyal; Martinelli, Fabio; Sly, Allan; Toninelli, Scaling limit and cube-root fluctuations in SOS surfaces above a wall, J. Eur. Math. Soc. (JEMS), 18, 5, 931-995 (2016) · Zbl 1344.60091

[12] Caputo, Pietro; Martinelli, Fabio; Toninelli, Entropic repulsion in \(|\nabla \phi |^p\) surfaces: a large deviation bound for all \(p\ge 1\), Boll. Un. Mat. Ital., 10, 3, 451-466 (2017) · Zbl 1375.60134

[13] Cesi, Filippo; Martinelli, Fabio, On the layering transition of an SOS surface interacting with a wall. I. Equilibrium results, J. Statist. Phys., 82, 3-4, 823-913 (1996) · Zbl 1042.82512

[14] Cesi, Filippo; Martinelli, Fabio, On the layering transition of an SOS surface interacting with a wall. II. The Glauber dynamics, Comm. Math. Phys., 177, 1, 173-201 (1996) · Zbl 0901.60076

[15] Chalker, The pinning of an interface by a planar defect, J. Phys. A, 15, 9, L481-L485 (1982)

[16] Coquille, Loren; Velenik, Yvan, A finite-volume version of Aizenman-Higuchi theorem for the 2d Ising model, Probab. Theory Related Fields, 153, 1-2, 25-44 (2012) · Zbl 1246.82013

[17] Dinaburg; Mazel, Layering transition in SOS model with external magnetic field, J. Statist. Phys., 74, 3-4, 533-563 (1994) · Zbl 0827.60099

[18] Dobrushin, Gibbs states describing a coexistence of phases for the three-dimensional Ising model, Theor. Probability Appl., 17, 4, 582-600 (1972) · Zbl 0275.60119

[19] von Dreifus, Henrique; Klein, Abel; Perez, Taming Griffiths' singularities: infinite differentiability of quenched correlation functions, Comm. Math. Phys., 170, 1, 21-39 (1995) · Zbl 0820.60086

[20] Fröhlich, Jürg; Spencer, Thomas, The Kosterlitz-Thouless transition in two-dimensional abelian spin systems and the Coulomb gas, Comm. Math. Phys., 81, 4, 527-602 (1981)

[21] Giacomin, Giambattista; Lacoin, Hubert, Disorder and wetting transition: the pinned harmonic crystal in dimension three or larger, Ann. Appl. Probab., 28, 1, 577-606 (2018) · Zbl 1388.60161

[22] Gouéré, Jean-Baptiste, Subcritical regimes in the Poisson Boolean model of continuum percolation, Ann. Probab., 36, 4, 1209-1220 (2008) · Zbl 1148.60077

[23] Hall, Peter, On continuum percolation, Ann. Probab., 13, 4, 1250-1266 (1985) · Zbl 0588.60096

[24] Higuchi, Yasunari, On some limit theorems related to the phase separation line in the two-dimensional Ising model, Z. Wahrsch. Verw. Gebiete, 50, 3, 287-315 (1979) · Zbl 0406.60084

[25] Holley, Richard, Remarks on the \({\rm FKG}\) inequalities, Comm. Math. Phys., 36, 227-231 (1974)

[26] Ioffe, D.; Velenik, Y., Low-temperature interfaces: prewetting, layering, faceting and Ferrari-Spohn diffusions, Markov Process. Related Fields, 24, 3, 487-537 (2018) · Zbl 1414.60079

[27] Kotecký, R.; Preiss, D., Cluster expansion for abstract polymer models, Comm. Math. Phys., 103, 3, 491-498 (1986) · Zbl 0593.05006

[28] Lacoin, Hubert, Wetting and layering for solid-on-solid I: Identification of the wetting point and critical behavior, Comm. Math. Phys., 362, 3, 1007-1048 (2018) · Zbl 1398.82023

[29] Swendsen, Roughening transition in the solid-on-solid model, Phys. Rev. B, 15, 2, 689-692 (1977)

[30] Temperley, Statistical mechanics and the partition of numbers. II. The form of crystal surfaces, Math. Proc. Cambridge Philos. Soc., 48, 683-697 (1952) · Zbl 0048.19802

[31] Weeks; Gilmer; Leamy, Structural Transition in the Ising-Model Interface, Phys. Rev. Lett., 31, 8, 549-551 (1973)