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Multi-colored links from 3-strand braids carrying arbitrary symmetric representations. (English) [Zbl 1426.81044](#)

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Summary: Obtaining HOMFLY-PT polynomials H_{R_1, \dots, R_l} for arbitrary links with l components colored by arbitrary $SU(N)$ representations R_1, \dots, R_l is a very complicated problem. For a class of rank r symmetric representations, the $[r]$ -colored HOMFLY-PT polynomial $H_{[r_1], \dots, [r_l]}$ evaluation becomes simpler, but the general answer lies far beyond our current capabilities. To simplify the situation even more, one can consider links that can be realized as a 3-strand closed braid. Recently [*H. Itoyama et al., Int. J. Mod. Phys. A* 28, No. 3-4, Paper No. 1340009, 81 p. (2013; [Zbl 1259.81082](#))], it was shown that $H_{[r]}$ for knots realized by 3-strand braids can be constructed using the quantum Racah coefficients (6j-symbols) of $U_q(\mathfrak{sl}_2)$, which makes easy not only to evaluate such invariants, but also to construct analytical formulas for $H_{[r]}$ of various families of 3-strand knots. In this paper, we generalize this approach to links whose components carry arbitrary symmetric representations. We illustrate the technique by evaluating multi-colored link polynomials $H_{[r_1], [r_2]}$ for the two-component link $L7a3$ whose components carry $[r_1]$ and $[r_2]$ colors. Using our results for exclusive Racah matrices, it is possible to calculate symmetric-colored HOMFLY-PT polynomials of links for the so-called one-looped links, which are obtained from arborescent links by adding a loop. This is a huge class of links that contains the entire Rolfsen table, all 3-strand links, all arborescent links, and, for example, all mutant knots with 11 intersections.

MSC:

- [81R50](#) Quantum groups and related algebraic methods applied to problems in quantum theory Cited in 1 Document
- [17B37](#) Quantum groups (quantized enveloping algebras) and related deformations
- [22E70](#) Applications of Lie groups to the sciences; explicit representations
- [58J28](#) Eta-invariants, Chern-Simons invariants
- [57K14](#) Knot polynomials

Full Text: [DOI](#)

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