

**Javili, Ali**

**Variational formulation of generalized interfaces for finite deformation elasticity.** (English)

Zbl 1425.74199

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Summary: The objective of this contribution is to formulate generalized interfaces in a variationally consistent manner within a finite deformation continuum mechanics setting. The general interface model is a zero-thickness model that represents the finite thickness “interphase” between different constituents in a heterogeneous material. The interphase may be the transition zone between inclusion and matrix in composites or the grain boundaries in polycrystalline solids. The term “general” indicates that the interface model here accounts for both jumps of the deformation as well as the traction across the interface. Both the cohesive zone model and elastic interface model can be understood as two limits of the current interface model. Furthermore, some aspects of material modeling of generalized interfaces are elaborated and a consistent model is proposed. Finally, the proposed theory is elucidated via a series of numerical examples.

Reviewer: [Reviewer \(Berlin\)](#)

**MSC:**

[74G65](#) Energy minimization in equilibrium problems in solid mechanics

[74B20](#) Nonlinear elasticity

Cited in **4** Documents

**Keywords:**

[general imperfect interface](#); [variational elasticity](#); [finite deformation](#); [size effect](#)

**Full Text:** [DOI](#)

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