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Geometrization of the generalized Fibonacci numeration system with applications to number theory. (Russian. English summary) Zbl 1428.11022

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Summary: Generalized Fibonacci numbers $\left\{F_i^{(g)}\right\}$ are defined by the recurrence relation

$$F_{i+2}^{(g)} = gF_{i+1}^{(g)} + F_i^{(g)}$$

with the initial conditions $F_0^{(g)} = 1$, $F_1^{(g)} = g$. These numbers generate representations of natural numbers as a greedy expansions

$$n = \sum_{i=0}^{k} \varepsilon_i(n) F_i^{(g)},$$

with natural conditions on $\varepsilon_i(n)$. In particular, when $g = 1$ we obtain the well-known Fibonacci numeration system. The expansions obtained by $g > 1$ are called representations of natural numbers in generalized Fibonacci numeration systems.

This paper is devoted to studying the sets $\mathbb{F}^{(g)}(\varepsilon_0, \ldots, \varepsilon_l)$, consisting of natural numbers with a fixed end of their representation in the generalized Fibonacci numeration system. The main result is the following geometrization theorem that describe the sets $\mathbb{F}^{(g)}(\varepsilon_0, \ldots, \varepsilon_l)$ in terms of the fractional parts of the form $\{n\tau_g\}$, $\tau_g = \frac{\sqrt{g^2+4}-g}{2}$.

More precisely, for any admissible ending $(\varepsilon_0, \ldots, \varepsilon_l)$ there exist effectively computable $a, b \in \mathbb{Z}$ such that $n \in \mathbb{F}^{(g)}(\varepsilon_0, \ldots, \varepsilon_l)$ if and only if the fractional part $\{(n+1)\tau_g\}$ belongs to the segment $[\{-a\tau_g\}; \{-b\tau_g\}]$. Earlier, a similar theorem was proved by the authors in the case of the classical Fibonacci numeration system.

As an application some analogues of classic number-theoretic problems for the sets $\mathbb{F}^{(g)}(\varepsilon_0, \ldots, \varepsilon_l)$ are considered. In particular asymptotic formulas for the quantity of numbers from the considered sets belonging to a given arithmetic progression, for the number of primes from the considered sets, for the number of representations of a natural number as a sum of a predetermined number of summands from the considered sets, and for the number of solutions of Lagrange, Goldbach and Hua Lookeng problem in the numbers of from the considered sets are established.

MSC:

11A67 Other number representations
11B39 Fibonacci and Lucas numbers and polynomials and generalizations

Keywords:

generalized Fibonacci numeration system; geometrization theorem; distribution in progressions; Goldbach type problem

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