Ramm, A. G.
Random fields estimation theory. (English) Zbl 0712.47042
Pitman Monographs and Surveys in Pure and Applied Mathematics, 48. Harlow: Longman Scientific & Technical; New York: John Wiley & Sons, Inc. vii, 274 p. £48.00 (1990).

The contents of this book represent a general survey of recent results on an application of a linear estimation problem including, in particular, contributions by the author.

Let $mv(f)$ represent the mean value of a function f on Euclidean space $\mathbb{R}^k$, where k is a positive integer (denoted in the book by $\bar{f}$). If A is a known operator, and $\mathcal{U}(x) = s(x) + n(x)$, $x \in \mathbb{R}^k$, is a random field on a domain D of $\mathbb{R}^k$, then a linear estimation problem relates to the determination of

$$(*1)\ \hat{\mathcal{U}}(x) = \int_D h(x,y)\mathcal{U}(y)dy,$$

such that

$$(*2)\ mv(|\hat{\mathcal{U}} - As|^2) \text{ is minimum,}$$

where h is defined as a distribution. By standard procedure, indicated in sections of the book in which the basic problem is formulated, it is shown that a necessary condition for a solution of (*1) is the equation

$$(*3)\ \int_D B(z,y)h(x,y)dy = f(z,x), \quad z,x \in \bar{D} = D \cup F,$$

where F is the 'smooth' boundary of bounded domain D, $f(y,x) = mv(\mathcal{U}^*(y)(As(x))) = f = *(x,y)$, a^* represents a complex conjugate of a, and B is the covariance function of the random field U and is also the space of test functions from which h is defined. Results of the book are stated for cases in which B is a kernel of a positive rational function of a selfadjoint elliptic operator L of order μ in $H = L^2(D)$ with spectrum Λ, spectral kernel $\phi(x,y,\lambda)$, and spectral measure $d\rho(\lambda)$, so that

$$(*4) \quad B(x,y) = \int_\lambda P(\lambda)Q^{-1}(\lambda)\phi(x,y,\lambda)d\rho(\lambda),$$

where $P(\lambda) > 0$, $Q(\lambda) > 0$, for $\lambda \in \Lambda$, P, Q are polynomials of degrees p, q respectively.

If $b = (q-p)s$, and the operator T is defined in terms of kernel B, so that T(h) is represented as f of (*3), then the basic theorem of the book indicates that $T: H^{-b}(D) \to H^b(D)$ is an isomorphism, and a solution of $T(h) = f$ with minimum order of singularity $ord(h) \leq b$ is derived as $h(x) = Q(L)G(x)$, where $G(x) = g(x) + v(x)$ in D, $G(x) = u(x)$ in $\mathbb{R}^k \setminus D$, $P(L)g = f$ in D, $P(L)v = 0$ in D, $Q(L)u = 0$ in $\mathbb{R}^k \setminus D$.

Proofs of the main results are provided in Chapter IV, and various applications indicated in Chapter V relate to linear estimation problems which arise in acoustics, geophysics, optics and estimation theory.

Although the main contents of the book deal especially with solutions of specified integral equations, familiarity with terms of optimization results, probability and statistics is required. Auxiliary results stated in Chapter VI of the book include most relevant definitions encountered in earlier chapters. An appendix indicates some connections between results of estimation and scattering theory and linear estimation problems.

Reviewer: G.O.Okikiolu

MSC:

47G10 Integral operators
60G60 Random fields
47-02 Research exposition (monographs, survey articles) pertaining to operator theory
60-02 Research exposition (monographs, survey articles) pertaining to probability theory

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