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A common generalization of functional equations characterizing normed and quasi-inner-product spaces. (English) [Zbl 0712.39021](#)
Can. Math. Bull. 35, No. 3, 321-327 (1992).

We determine the general solutions of the functional equation $f_1(x+y) + f_2(x-y) = f_3(x) + f_4(y)$, $x, y \in G$ for $f_i : G \rightarrow F$ ($i = 1, 2, 3, 4$), where G is a 2-divisible group and F is a commutative field of characteristic different from 2. The motivation for studying this equation came from a result due to Drygas where he proved a Jordan and von Neumann type characterization theorem for quasi-inner products. Also, this equation is a generalization of the quadratic functional equation investigated by several authors in connection with inner product spaces and their generalizations. Special cases of this equation include the Cauchy equation, the Jensen equation, the Pexider equation and many more. Here, we determine the general solution of this equation without any regularity assumptions on f_i .

Reviewer: P.K.Sahoo

MSC:

[39B42](#) Matrix and operator functional equations

[39B52](#) Functional equations for functions with more general domains and/or ranges

[46C15](#) Characterizations of Hilbert spaces

Cited in **2** Reviews
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Keywords:

homomorphism; bihomomorphism; characterization; quasi-inner products; quadratic functional equation; inner product spaces; Cauchy equation; Jensen equation; Pexider equation; general solution

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