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**Decomposition of Brownian loop-soup clusters.** (English) Zbl 07117731  
J. Eur. Math. Soc. (JEMS) 21, No. 10, 3225-3253 (2019).

**Summary:** We study the structure of Brownian loop-soup clusters in two dimensions. Among other things, we obtain the following decomposition of the clusters with critical intensity: If one conditions a loop-soup cluster on its outer boundary  $\partial$  (which is known to be an  $SLE_4$ -type loop), then the union of all excursions away from  $\partial$  by all the Brownian loops in the loop-soup that touch  $\partial$  is distributed exactly like the union of all excursions of a Poisson point process of Brownian excursions in the domain enclosed by  $\partial$ .

A related result that we derive and use is that the couplings of the Gaussian Free Field (GFF) with  $CLE_4$  via level lines (by Miller-Sheffield), of the square of the GFF with loop-soups via occupation times (by Le Jan), and of the  $CLE_4$  with loop-soups via loop-soup clusters (by Sheffield and Werner) can be made to coincide. An instrumental role in our proof of this fact is played by Lupu's description of  $CLE_4$  as limits of discrete loop-soup clusters.

**MSC:**

[60J65](#) Brownian motion  
[60J67](#) Stochastic (Schramm-)Loewner evolution (SLE)  
[60G60](#) Random fields

**Keywords:**

[Brownian loop-soups](#); [Schramm-Loewner evolutions](#)

**Full Text:** [DOI](#) [arXiv](#)

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