Seppälä, Louna
Euler's factorial series at algebraic integer points. (English) Zbl 1437.11095
J. Number Theory 206, 250-281 (2020).

Let $\mathbb{K}$ be a number field, n be a positive integer and $\alpha_1, \dots, \alpha_n \in \mathbb{Z}_{\mathbb{K}} \setminus \{0\}$. Let $\lambda_0, \dots \lambda_n \in \mathbb{Z}_{\mathbb{K}}$ not all equal zero. Assume that V be a non-empty collection of non-Archimedian valuations of $\mathbb{K}$. Then under the special conditions, the author proves that there exists valuation $v \in V$ such that $\lambda_0 + \sum_{j=1}^{n} \lambda_j F_v(\alpha_j) \neq 0$ where $F_v(t) = \sum_{i=0}^{\infty} i! t^i$ is a series as a function in v-adic domain $\mathbb{K}$. In addition, if H is a real number such that $H \geq \prod_{w \in V_\infty} \max_{0 \leq i \leq n}\{\|\lambda_i\|_w\}$ then under other special conditions there exists prime p with $p \in] \log(\frac{\log H}{\log \log H}), \frac{17n \log H}{\log \log H}[$ and valuation $v|p$ such that

$$\|\lambda_0 + \sum_{j=1}^{n} \lambda_j F_v(\alpha_j)\|_v > H^{-(n+1)-114n^2 \frac{\log \log \log H}{\log \log H}}.$$

The proofs use the method of Padé approximation.

Reviewer: Jaroslav Hančl (Ostrava)

MSC:

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41A21 Padé approximation

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Diophantine approximation; divergent series; number field; Padé approximation; p-adic valuation

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