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A stabilized RBF finite difference method for convection dominated flows over meshfree nodes. (English) Zbl 07110384

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Summary: In this paper, a stabilized solution scheme is presented for solving highly convective flow equations on meshfree nodal distribution. The stabilized terms are obtained by considering higher order approximation of governing differential equations over finite control volume while applying force and momentum balance. Spatial derivatives, of resulting flow equations, are treated with Radial Basis Functions in Finite Difference Method (RBF-FD) over meshfree nodal cloud. The characteristic length, for applying equilibrium of forces and momentum, is proposed to be a function of Reynolds number and flow velocity. Performance and accuracy of the proposed scheme is tested for 1-D Convection-Diffusion equations. Numerical tests are conducted for initial conditions having step as well as uniformly varying field variables. The scheme is found to be effective in suppressing the non-physical numerical fluctuations associated with convection dominated flows. Accuracy of the solution with stabilized term is found to be higher than the one without stabilization. The solution scheme is also used for flow around static NACA0012 airfoil at $\mathrm{Re} = 10,000$. The stabilization term is found to effectively suppress the numerical oscillation when compared to non-stabilized scheme.

MSC:

76 Fluid mechanics
74 Mechanics of deformable solids

Keywords:

flow stabilization; RBF-FD; meshfree methods; hybrid grid; meshfree methods

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References:

[1] Chew, C. S.; Yeo, K. S.; Shu, C., A generalized finite-difference (GFD) ALE scheme for incompressible flows around moving solid bodies on hybrid meshfree-cartesian grids, J Comput Phys, 218, 2, 510-548, (2006) · Zbl 1161.76525

[2] Ding, H.; Shu, C.; Yeo, K. S.; Xu, D., Simulation of incompressible viscous flows past a circular cylinder by hybrid FD scheme and meshless least square-based finite difference method, Comput Methods Appl Mech Eng, 193, 9-11, 727-744, (2004) · Zbl 1068.76062

[3] Chinchapatnam, P. P.; Djidjeli, K.; Nair, P. B., Radial basis function meshless method for the steady incompressible Navier-Stokes equations, Int J Comput Math, 84, 10, 1509-1521, (2007) · Zbl 1123.76048

[4] Sanyasiraju, Y.; Chandhini, G., Local radial basis function based gridfree scheme for unsteady incompressible viscous flows, J Comput Phys, 227, 20, 8922-8948, (2008) · Zbl 1146.76045

[5] Wang, J. G.; Liu, G. R., On the optimal shape parameters of radial basis functions used for 2-D meshless methods, Comput Methods Appl Mech Eng, 191, 23-24, 2611-2630, (2002) · Zbl 1065.74074

[6] Shu, H. D.C.; Yeo, K. S., Local radial basis function-based differential quadrature method and its application to solve two-dimensional incompressible Navier-Stokes equations, Comput Methods Appl Mech Eng, 192, 7-8, 941-954, (2003) · Zbl 1025.76036

[7] Shu, H. D.C.; Yeo, K. S., Computation of incompressible Navier-Stokes equations by local RBF-based differential quadrature method, Comput Model Eng Sci, 7, 2, 195-206, (2005) · Zbl 1106.76427

[8] Hardy, R. L., Multiquadric equations of topography and other irregular surfaces, J Geophys Res, 76, 8, 1905, (1971)

[9] Kansa, E. J., Multiquadrics – a scattered data approximation scheme with applications to computational fluid-dynamics-II solutions to parabolic, hyperbolic and elliptic partial-differential equations, Comput Math Appl, 19, 8-9, 147-161, (1990) · Zbl 0850.76048

[10] Franke, C.; Schaback, R., Solving partial differential equations by collocation using radial basis functions, Appl Math Comput, 93, 1, 73-82, (1998) · Zbl 0943.65133

[11] Larsson, E.; Fornberg, B., Theoretical and computational aspects of multivariate interpolation with increasingly flat radial basis functions, Comput Math Appl, 49, 1, 103-130, (2005) · Zbl 1074.41012

[12] Chen, C. S.; Brebbia, C. A., The dual reciprocity method for Helmholtz-type operators, WIT Trans. Model. Simul., 21, (1998) · Zbl 0929.65109

[13] Hon, Y.-C.; Šarler, B.; Yun, D.-f., Local radial basis function collocation method for solving thermo-driven fluid-flow problems with free surface, Eng Anal Bound Elem, 57, 2-8, (2015) · Zbl 1403.76140

[14] Mai-Duy, N.; Tran-Cong, T., Numerical solution of differential equations using multiquadric radial basis function networks, Neural Netw, 14, 2, 185-199, (2001)

[15] Driscoll, T. A.; Fornberg, B., Interpolation in the limit of increasingly flat radial basis functions, Comput Math Appl, 43, 3-5, 413-422, (2002) · Zbl 1006.65013

[16] Chen, W.; Tanaka, M., A meshless, integration-free, and boundary-only RBF technique, Comput Math Appl, 43, 3-5, 379-391, (2002) · Zbl 0999.65142

[17] Tolstykh, A. I.; Shirobokov, D. A., On using radial basis functions in a "finite difference mode" with applications to elasticity problems, Comput Mech, 33, 1, 68-79, (2003) · Zbl 1063.74104

[18] Wright, G. B.; Fornberg, B., Scattered node compact finite difference-type formulas generated from radial basis functions, J Comput Phys, 212, 1, 99-123, (2006) · Zbl 1089.65020

[19] Liu, G.; Zhang, J.; Li, H.; Lam, K.; Kee, B. B., Radial point interpolation based finite difference method for mechanics problems, Int J Numer Methods Eng, 68, 7, 728-754, (2006) · Zbl 1129.74049

[20] Chinchapatnam, P. P.; Djidjeli, K.; Nair, P.; Tan, M., A compact RBF-FD based meshless method for the incompressible Navier-Stokes equations, Proc Inst Mech Eng Part M J Eng Marit Environ, 223, 3, 275-290, (2009)

[21] Hirsch, C., Numerical computation of internal and external flows, Computational methods for inviscid and viscous flows, 2, (2002), Wiley-Interscience Publications

[22] Brooks, A. N.; Hughes, T. J., Streamline upwind/Petrov-Galerkin formulations for convection dominated flows with particular emphasis on the incompressible Navier-Stokes equations, Comput Methods Appl Mech Eng, 32, 1-3, 199-259, (1982) · Zbl 0497.76041

[23] Gu, Y. T.; Liu, G. R., A boundary point interpolation method for stress analysis of solids, Comput Mech, 28, 1, 47-54, (2002) · Zbl 1115.74380

[24] Kee, B. B.; Liu, G.; Lu, C., A regularized least-squares radial point collocation method (RLS-RPCM) for adaptive analysis, Comput Mech, 40, 5, 837-853, (2007) · Zbl 1166.74051

[25] Liu, G.; Kee, B. B.; Chun, L., A stabilized least-squares radial point collocation method (LS-RPCM) for adaptive analysis, Comput Methods Appl Mech Eng, 195, 37-40, 4843-4861, (2006) · Zbl 1128.74050

[26] Rabczuk, T.; Belytschko, T.; Xiao, S., Stable particle methods based on lagrangian kernels, Comput Methods Appl Mech Eng, 193, 12-14, 1035-1063, (2004) · Zbl 1060.74672

[27] Rabczuk, T.; Belytschko, T., A three-dimensional large deformation meshfree method for arbitrary evolving cracks, Comput Methods Appl Mech Eng, 196, 29-30, 2777-2799, (2007) · Zbl 1128.74051

[28] Fornberg, B.; Lehto, E., Stabilization of RBF-generated finite difference methods for convective PDEs, J Comput Phys, 230, 6, 2270-2285, (2011) · Zbl 1210.65154

[29] Shen, Q., Local RBF-based differential quadrature collocation method for the boundary layer problems, Eng Anal Bound Elem, 34, 3, 213-228, (2010) · Zbl 1244.65118

[30] Chan, Y.; Shen, L.; Wu, C.; Young, D., A novel upwind-based local radial basis function differential quadrature method for convection-dominated flows, Comput Fluids, 89, 157-166, (2014) · Zbl 1391.76529

[31] Shu, C.; Ding, H.; Chen, H.; Wang, T., An upwind local RBF-DQ method for simulation of inviscid compressible flows, Comput Methods Appl Mech Eng, 194, 18, 2001-2017, (2005) · Zbl 1093.76052

[32] Oñate, E., Derivation of stabilized equations for numerical solution of advective-diffusive transport and fluid flow problems, Comput Methods Appl Mech Eng, 151, 1, 233-265, (1998) · Zbl 0916.76060

[33] Oñate, E.; Idelsohn, S.; Zienkiewicz, O. C.; Taylor, R. L.; Sacco, C., A stabilized finite point method for analysis of fluid mechanics problems, Comput Methods Appl Mech Eng, 139, 1-4, 315-346, (1996) · Zbl 0894.76065

[34] Javed, A.; Djijdeli, K.; Xing, T.; Jing, Shape adaptive RBF-FD implicit scheme for incompressible viscous Navier-Strokes equations, Comput Fluids, 89, 0, 38-52, (2014) · Zbl 1391.76479

[35] Javed, A.; Djidjeli, K.; Xing, J. T.; Sun, Z., An ALE based hybrid meshfree local RBF-Cartesian FD scheme for incompressible flow around moving boundaries, Proceedings of the forty fourth AIAA fluid dynamics conference. AIAA aviation, 2312, (2014), American Institute of Aeronautics and Astronautics

[36] Akbari, M.; Price, S., Simulation of dynamic stall for a NACA 0012 airfoil using a vortex method, J Fluids Struct, 17, 6, 855-874, (2003)

[37] Alam, M. M.; Zhou, Y.; Yang, H.; Guo, H.; Mi, J., The ultra-low Reynolds number airfoil wake, Exp Fluids, 48, 1, 81-103, (2010)