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Contractive linear preservers of absolutely compatible pairs between C^* -algebras. (English)

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Summary: Let a and b be elements in the closed ball of a unital C^* -algebra A (if A is not unital we consider its natural unitization). We shall say that a and b are domain (respectively, range) absolutely compatible ($a\Delta_d b$, respectively, $a\Delta_r b$, in short) if $\| |a| - |b| \| + \| 1 - |a| - |b| \| = 1$ (respectively, $\| |a^*| - |b^*| \| + \| 1 - |a^*| - |b^*| \| = 1$), where $|a|^2 = a^*a$. We shall say that a and b are absolutely compatible ($a\Delta b$ in short) if they are both range and domain absolutely compatible. In general, $a\Delta_d b$ (respectively, $a\Delta_r b$ and $a\Delta b$) is strictly weaker than $ab^* = 0$ (respectively, $a^*b = 0$ and $a \perp b$). Let $T : A \rightarrow B$ be a non-expansive bounded linear mapping between C^* -algebras. We prove that, if T preserves domain absolutely compatible elements (i.e., $a\Delta_d b \Rightarrow T(a)\Delta_d T(b)$), then T is a triple homomorphism. A similar statement is proved when T preserves range absolutely compatible elements. It is finally shown that T is a triple homomorphism if, and only if, T preserves absolutely compatible elements.

MSC:

46L05 General theory of C^* -algebras

47B48 Linear operators on Banach algebras

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