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Towards three-dimensional conformal probability. (English) Zbl 1420.81023
p-Adic Numbers Ultrametric Anal. Appl. 10, No. 4, 233-252 (2018).

Summary: In this outline of a program, based on rigorous renormalization group theory, we introduce new definitions which allow one to formulate precise mathematical conjectures related to conformal invariance as studied by physicists in the area known as higher-dimensional conformal bootstrap which has developed at a breathtaking pace over the last few years. We also explore a second theme, intimately tied to conformal invariance for random distributions, which can be construed as a search for very general first and second-quantized Kolmogorov-Chentsov Theorems. First-quantized refers to regularity of sample paths. Second-quantized refers to regularity of generalized functionals or Hida distributions and relates to the operator product expansion. We formulate this program in both the Archimedean and *p*-adic situations. Indeed, the study of conformal field theory and its connections with probability provides a golden opportunity where *p*-adic analysis can lead the way towards a better understanding of open problems in the Archimedean setting. Finally, we present a summary of progress made on a *p*-adic hierarchical model and point out possible connections to number theory. Parts of this article were presented in author's talk at the 6th International Conference on *p*-adic Mathematical Physics and its Applications, Mexico 2017.

MSC:

- 81T40** Two-dimensional field theories, conformal field theories, etc. in quantum mechanics Cited in 4 Documents
- 81T17** Renormalization group methods applied to problems in quantum field theory
- 81T10** Model quantum field theories
- 81T70** Quantization in field theory; cohomological methods
- 11K41** Continuous, *p*-adic and abstract analogues

Keywords:

conformal field theory; *p*-adic quantum field theory; scaling limits; renormalization group

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References:

- [1] A. Abdesselam, *Renormalisation Constructive Explicite*, Ph.D. Thesis, école Polytechnique (1997). Available in part at <http://people.virginia.edu/~aa4cr/these.pdf>.
- [2] Abdesselam, A., A complete renormalization group trajectory between two fixed points, *Comm. Math. Phys.*, 276, 727-772, (2007) · [Zbl 1194.81168](#)
- [3] Abdesselam, A.; Exner, P. (ed.), Renormalization group trajectories between two fixed points, 346-351, (2010) · [Zbl 1203.81122](#)
- [4] Abdesselam, A., A massless quantum field theory over the *p*-adics, *Oberwolfach Reports*, 8, 781-824, (2011)
- [5] A. Abdesselam, "QFT, RG, and all that, for mathematicians," eleven pages. [arXiv:1311.4897\[math.PR\]](#) (2013).
- [6] A. Abdesselam, "Proof of a 43-year-old prediction by Wilson on anomalous scaling for a hierarchical composite field," Slides of 06/11/2015 talk given at the conference *Constructive Renormalization Group: A Conference in Memory of Pierluigi Falco*, Roma, Italy. Available at <http://people.virginia.edu/~aa4cr/FalcoConferenceTalk11June2015.pdf>.
- [7] A. Abdesselam, "A second-quantized Kolmogorov-Chentsov theorem," [arXiv:1604.05259\[math.PR\]](#) (2016).
- [8] A. Abdesselam, A. Chandra and G. Guadagni, "Rigorous quantum field theory functional integrals over the *p*-adics: research announcement," [arXiv:1210.7717\[math.PR\]](#) (2012).
- [9] A. Abdesselam, A. Chandra and G. Guadagni, "Rigorous quantum field theory functional integrals over the *p*-adics I: anomalous dimensions," [arXiv:1302.5971\[math.PR\]](#) (2013).

- [10] A. Abdesselam, A. Chandra and G. Guadagni, “Rigorous quantum field theory functional integrals over the p-adics II: full scale invariance,” in preparation.
- [11] R. J. Adler and J. E. Taylor, *Random Fields and Geometry*, Springer Monographs in Mathematics (Springer, New York, 2007). · [Zbl 1149.60003](#)
- [12] Ahlfors, L. V.; Hersch, J. (ed.); Huber, A. (ed.), *Cross-ratios and Schwarzian derivatives in \mathbb{R}^n* , 1-15, (1988), Basel
- [13] Aizenbud, A.; Gourevitch, D., Schwartz functions on Nash manifolds, *Int. Math. Res. Not. IMRN*, 5, 37, (2008) · [Zbl 1161.58002](#)
- [14] Aizenman, M.; Fernández, R., Critical exponents for long-range interactions, *Lett. Math. Phys.*, 16, 39-49, (1988) · [Zbl 0658.60136](#)
- [15] S. Albeverio, A. Yu. Khrennikov and V. M. Shelkovich, *Theory of p-Adic Distributions: Linear and Nonlinear Models*, London Mathematical Society Lecture Note Series **370** (Cambridge Univ. Press, Cambridge, 2010). · [Zbl 1198.46001](#)
- [16] Allez, R.; Rhodes, R.; Vargas, V., Lognormal \ast -scale invariant random measures, *Probab. Theory Related Fields*, 155, 751-788, (2013) · [Zbl 1278.60083](#)
- [17] Angelini, M. C.; Parisi, G.; Ricci-Tersenghi, F., Relations between short-range and long-range Ising models, *Phys. Rev. E*, 89, 062120, (2014)
- [18] H. Bahouri, J.-Y. Chemin and R. Danchin, *Fourier Analysis and Nonlinear Partial Differential Equations*, Grundlehren der Mathematischen Wissenschaften **343** (Springer, Heidelberg, 2011).
- [19] Bargetz, C., Explicit representations of spaces of smooth functions and distributions, *J. Math. Anal. Appl.*, 424, 1491-1505, (2015) · [Zbl 1321.46025](#)
- [20] Bargmann, V., Irreducible unitary representations of the Lorentz group, *Ann. Math. (2)*, 48, 568-640, (1947) · [Zbl 0045.38801](#)
- [21] Bauerschmidt, R.; Brydges, D. C.; Slade, G., Scaling limits and critical behaviour of the 4-dimensional \mathbb{S}^4 -component $|\varphi|^4$ spin model, *J. Stat. Phys.*, 157, 692-742, (2014) · [Zbl 1308.82026](#)
- [22] Bauerschmidt, R.; Brydges, D. C.; Slade, G., Structural stability of a dynamical system near a nonhyperbolic fixed point, *Ann. Henri Poincaré*, 16, 1033-1065, (2015) · [Zbl 1347.37041](#)
- [23] Bauerschmidt, R.; Brydges, D. C.; Slade, G., A renormalisation group method. III. Perturbative analysis, *J. Stat. Phys.*, 159, 492-529, (2015) · [Zbl 1319.82008](#)
- [24] Bauerschmidt, R.; Brydges, D. C.; Slade, G., Logarithmic correction for the susceptibility of the 4-dimensional weakly self-avoiding walk: a renormalisation group analysis, *Comm. Math. Phys.*, 337, 817-877, (2015) · [Zbl 1318.60049](#)
- [25] Bauerschmidt, R.; Brydges, D. C.; Slade, G., Critical two-point function of the 4-dimensional weakly self-avoiding walk, *Comm. Math. Phys.*, 338, 169-193, (2015) · [Zbl 1320.82031](#)
- [26] Becnel, J. J., Equivalence of topologies and Borel fields for countably-Hilbert spaces, *Proc. Amer. Math. Soc.*, 134, 581-590, (2006) · [Zbl 1081.57019](#)
- [27] Behan, C.; Rastelli, L.; Rychkov, S.; Zan, B., A scaling theory for the long-range to short-range crossover and an infrared duality, *J. Phys. A*, 50, 48, (2017) · [Zbl 1376.82012](#)
- [28] Belavin, A. A.; Polyakov, A. M.; Zamolodchikov, A. B., Infinite conformal symmetry of critical fluctuations in two dimensions, *J. Statist. Phys.*, 34, 763-774, (1984)
- [29] Belavin, A. A.; Polyakov, A. M.; Zamolodchikov, A. B., Infinite conformal symmetry in two-dimensional quantum field theory, *Nucl. Phys. B*, 241, 333-380, (1984) · [Zbl 0661.17013](#)
- [30] P. Billingsley, *Convergence of Probability Measures*, Second ed., Wiley Series in Probability and Statistics: Probability and Statistics. A Wiley-Interscience Publication, (John Wiley & Sons, Inc., New York, 1999). · [Zbl 0944.60003](#)
- [31] Blanchard, T.; Picco, M.; Rajabpour, M. A., Influence of long-range interactions on the critical behavior of the Ising model, *Europhysics Lett.*, 101, 56003, (2013)
- [32] Bleher, P. M.; Missarov, M. D., The equations of Wilson’s renormalization group and analytic renormalization. I. General results, *Comm. Math. Phys.*, 74, 235-254, (1980)
- [33] Bleher, P. M.; Missarov, M. D., The equations of Wilson’s renormalization group and analytic renormalization. II. Solution of Wilson’s equations, *Comm. Math. Phys.*, 74, 255-272, (1980)
- [34] Bleher, P. M.; Sinai, Ja. G., Investigation of the critical point in models of the type of Dyson’s hierarchical models, *Comm. Math. Phys.*, 33, 23-42, (1973)
- [35] Bleher, P. M.; Sinai, Ya. G., Critical indices for Dyson’s asymptotically-hierarchical models, *Comm. Math. Phys.*, 45, 247-278, (1975)
- [36] Blumenthal, R. M.; Gettoor, R. K., Some theorems on stable processes, *Trans. Amer. Math. Soc.*, 95, 263-273, (1960) · [Zbl 0107.12401](#)
- [37] Boer, J.; Verlinde, E.; Verlinde, H., On the holographic renormalization group, *J. High Energy Phys.*, 8, 3, (2000) · [Zbl 0989.81538](#)
- [38] V. I. Bogachev, *Gaussian Measures*. Mathematical Surveys and Monographs **62** (American Math. Society, Providence, RI, 1998).
- [39] N. N. Bogolubov, A. A. Logunov, A. I. Oksak and I. T. Todorov, *General Principles of Quantum Field Theory*, Translated from the Russian by G. G. Gould. *Mathematical Physics and Applied Mathematics* **10** (Kluwer Academic Publ. Group, Dordrecht, 1990).
- [40] Breuer, P.; Major, P., Central limit theorems for nonlinear functionals of Gaussian fields, *J. Multivariate Anal.*, 13, 425-441, (1983) · [Zbl 0518.60023](#)

- [41] Brezin, E.; Parisi, G.; Ricci-Tersenghi, F., The crossover region between long-range and short-range interactions for the critical exponent, *J. Stat. Phys.*, 157, 855-868, (2014) · [Zbl 1318.82017](#)
- [42] Brouder, C.; Dang, N. V.; Hélein, F., A smooth introduction to the wavefront set, *J. Phys. A*, 47, 443001, (2014) · [Zbl 1316.35003](#)
- [43] Bruhat, F., Distributions sur un groupe localement compact et applications à l'étude des représentations des groupes p-adiques, *Bull. Soc. Math. France*, 89, 43-75, (1961) · [Zbl 0128.35701](#)
- [44] Brydges, D., The renormalization group and self-avoiding walk, *Random Walks, Random Fields, and Disordered Systems*, 2144, 65-116, (2015) · [Zbl 1432.82015](#)
- [45] Brydges, D.; Dimock, J.; Hurd, T. R., The short distance behavior of $(\phi^4)_3$, *Comm. Math. Phys.*, 172, 143-186, (1995) · [Zbl 0858.60095](#)
- [46] Brydges, D. C.; Fröhlich, J.; Sokal, A. D., A new proof of the existence and nontriviality of the continuum ϕ^4 and ϕ^3 quantum field theories, *Comm. Math. Phys.*, 91, 141-186, (1983)
- [47] Brydges, D. C.; Mitter, P. K.; Scoppola, B., Critical $(\Phi^4)_\epsilon$, *Comm. Math. Phys.*, 240, 281-327, (2003) · [Zbl 1053.81065](#)
- [48] Brydges, D. C.; Slade, G., A renormalisation group method. I. Gaussian integration and normed algebras, *J. Stat. Phys.*, 159, 421-460, (2015) · [Zbl 1317.82013](#)
- [49] Brydges, D. C.; Slade, G., A renormalisation group method. II. Approximation by local polynomials, *J. Stat. Phys.*, 159, 461-491, (2015) · [Zbl 1317.82014](#)
- [50] Brydges, D. C.; Slade, G., A renormalisation group method. IV. Stability analysis, *J. Stat. Phys.*, 159, 530-588, (2015) · [Zbl 1317.82015](#)
- [51] Brydges, D. C.; Slade, G., A renormalisation group method. V. A single renormalisation group step, *J. Stat. Phys.*, 159, 589-667, (2015) · [Zbl 1317.82016](#)
- [52] J.-F. Burnol, "The explicit formula and a propagator," Preprint [arXiv:math/9809119\[math.NT\]](#), (1998).
- [53] J.-F. Burnol, "The explicit formula in simple terms," Preprint [arXiv:math/9810169\[math.NT\]](#), (1998).
- [54] Caffarelli, L.; Silvestre, L., An extension problem related to the fractional Laplacian, *Comm. Partial Diff. Equat.*, 32, 1245-1260, (2007) · [Zbl 1143.26002](#)
- [55] Camia, F.; Garban, C.; Newman, C., Planar Ising magnetization field I. Uniqueness of the critical scaling limit, *Ann. Probab.*, 43, 528-571, (2015) · [Zbl 1332.82012](#)
- [56] R. Catellier and K. Chouk, "Paracontrolled distributions and the 3-dimensional stochastic quantization equation," Preprint [arXiv:1310.6869\[math.PR\]](#), (2013). · [Zbl 1433.60048](#)
- [57] A. Chandra, *Construction and Analysis of a Hierarchical Massless Quantum Field Theory*, Ph.D. Thesis (University of Virginia, 2014). Available at <http://libra.virginia.edu/catalog/libra-oa:7070>.
- [58] Chandra, A.; Weber, H., Stochastic PDEs, regularity structures, and interacting particle systems, *Ann. Fac. Sci. Toulouse Math.* (6), 26, 847-909, (2017) · [Zbl 1421.81001](#)
- [59] Chelkak, D.; Hongler, C.; Izyurov, K., Conformal invariance of spin correlations in the planar Ising model, *Ann. Math.* (2), 181, 1087-1138, (2015) · [Zbl 1318.82006](#)
- [60] Chen, T.; Fröhlich, J.; Pizzo, A., Infraparticle scattering states in non-relativistic QED. I. The Bloch-Nordsieck paradigm, *Comm. Math. Phys.*, 294, 761-825, (2010) · [Zbl 1208.81211](#)
- [61] Connes, A., Trace formula in noncommutative geometry and the zeros of the Riemann zeta function, *Selecta Math.* (N.S.), 5, 29-106, (1999) · [Zbl 0945.11015](#)
- [62] G. Da Prato and L. Tubaro, "Wick powers in stochastic PDEs: an introduction," Preprint, (2007). Available at <http://eprints.biblio.unitn.it/1189/>.
- [63] Dang, N. V., The extension of distributions on manifolds, a microlocal approach, *Ann. Henri Poincaré*, 17, 819-859, (2016) · [Zbl 1337.81098](#)
- [64] Daubechies, I.; Grossmann, A.; Meyer, Y., Painless nonorthogonal expansions, *J. Math. Phys.*, 27, 1271-1283, (1986) · [Zbl 0608.46014](#)
- [65] Delamotte, B.; Tissier, M.; Wschebor, N., Scale invariance implies conformal invariance for the threedimensional Ising model, *Phys. Rev. E*, 93, 012144, (2016)
- [66] Deninger, C., Number theory and dynamical systems on foliated spaces, *Jahresber. Deutsch. Math.-Verein.*, 103, 79-100, (2001) · [Zbl 1003.11029](#)
- [67] De Roeck, W.; Kupiainen, A., Approach to ground state and time-independent photon bound for massless spin-boson models, *Ann. Henri Poincaré*, 14, 253-311, (2013) · [Zbl 1266.81163](#)
- [68] P. Di Francesco, P. Mathieu and D. Sénéchal, *Conformal Field Theory*, Graduate Texts in Contemporary Physics (Springer-Verlag, New York, 1997). · [Zbl 0869.53052](#)
- [69] Dobrushin, R. L., Gaussian and their subordinated self-similar random generalized fields, *Ann. Probab.*, 7, 1-28, (1979) · [Zbl 0392.60039](#)
- [70] Dobrushin, R. L., Automodel generalized random fields and their renorm group, *Multicomponent Random Systems*, 6, 153-198, (1980) · [Zbl 0499.60047](#)
- [71] Dolan, F. A.; Osborn, H., Conformal four point functions and the operator product expansion, *Nucl. Phys. B*, 599, 459-496, (2001) · [Zbl 1097.81734](#)

- [72] Dolan, F. A.; Osborn, H., Conformal partial waves and the operator product expansion, Nucl. Phys. B, 678, 491-507, (2004) · [Zbl 1097.81735](#)
- [73] Dragovich, B.; Khrennikov, A. Yu.; Kozyrev, S. V.; Volovich, I. V.; Zelenov, E. I., p-Adic mathematical physics: the first 30 years, p-Adic Numbers Ultrametric Anal. Appl., 9, 87-121, (2017) · [Zbl 1394.81009](#)
- [74] Drummond, I. T.; Shore, G. M., Conformal anomalies for interacting scalar fields in curved spacetime, Phys. Rev. D, 19, 1134-1143, (1979)
- [75] J. Dubédat, "Exact bosonization of the Ising model," Preprint arXiv:1112.4399[math.PR], (2011).
- [76] Dunau, J.-L.; Sénateur, H., Une caractérisation du type de la loi de Cauchy-conforme sur \mathbb{R}^n , Probab. Theory Rel. Fields, 77, 129-135, (1988) · [Zbl 0662.62008](#)
- [77] B. Duplantier, R. Rhodes, S. Sheffield and V. Vargas, "Log-correlated Gaussian fields: an overview," Preprint arXiv:1407.5605[math.PR], (2014). · [Zbl 1366.60023](#)
- [78] Dütsch, M.; Rehren, K.-H., A comment on the dual field in the AdS-CFT correspondence, Lett.Math. Phys., 62, 171-184, (2002) · [Zbl 1031.81055](#)
- [79] Dütsch, M.; Rehren, K.-H., Generalized free fields and the AdS-CFT correspondence, Ann. Henri Poincaré, 4, 613-635, (2003) · [Zbl 1038.81051](#)
- [80] Dybalski, W., Towards a construction of inclusive collision cross-sections in the massless Nelson model, Ann. Henri Poincaré, 13, 1427-1449, (2012) · [Zbl 1252.81121](#)
- [81] Dyson, F. J., Existence of a phase-transition in a one-dimensional Ising ferromagnet, Comm. Math. Phys., 12, 91-107, (1969) · [Zbl 1306.47082](#)
- [82] Dyson, F. J., An Ising ferromagnet with discontinuous long-range order, Comm. Math. Phys., 21, 269-283, (1971)
- [83] El-Showk, S.; Paulos, M. F.; Poland, D.; Rychkov, S.; Simmons-Duffin, D.; Vichi, A., Solving the 3D Ising model with the conformal bootstrap, Phys. Rev. D, 86, 025022, (2012) · [Zbl 1310.82013](#)
- [84] Falco, P., Kosterlitz-Thouless transition line for the two dimensional Coulomb gas, Comm. Math. Phys., 312, 559-609, (2012) · [Zbl 1254.82012](#)
- [85] P. Falco, "Critical exponents of the two dimensional Coulomb gas at the Berezinskii-Kosterlitz-Thouless transition," Preprint arXiv:1311.2237[math-ph], (2013).
- [86] Fernique, X., Processus linéaires, processus généralisés, Ann. Inst. Fourier (Grenoble), 17, 1-92, (1967) · [Zbl 0167.16702](#)
- [87] Fisher, M. E.; Ma, S.-K.; Nickel, B. G., Critical exponents for long-range interactions, Phys. Rev. Lett., 29, 917-920, (1972)
- [88] Föllmer, H., Calcul d'Itô sans probabilités, Séminaire de Probabilités XV, 850, 143-150, (1981)
- [89] Frank, R. L.; Lieb, E. H., Inversion positivity and the sharp Hardy-Littlewood-Sobolev inequality, Calc. Var. Partial Diff. Equat., 39, 85-99, (2010) · [Zbl 1204.39024](#)
- [90] Frazier, M.; Jawerth, B., A discrete transform and decompositions of distribution spaces, J. Funct. Anal., 93, 34-170, (1990) · [Zbl 0716.46031](#)
- [91] Frenkel, E.; Cartier, P. (ed.); Julia, B. (ed.); Moussa, P. (ed.); Vanhove, P. (ed.), Lectures on the Langlands program and conformal field theory, 387-533, (2007), Berlin
- [92] Fröhlich, J.; Simon, B.; Spencer, T., Infrared bounds, phase transitions and continuous symmetry breaking, Comm. Math. Phys., 50, 79-95, (1976)
- [93] Furlan, M.; Mourrat, J.-C., A tightness criterion for random fields, with application to the Ising model, Electron. J. Probab., 22, 97, (2017) · [Zbl 1378.60065](#)
- [94] Gawędzki, K.; Kupiainen, A., Block spin renormalization group for dipole gas and $(\mathbb{R}^n \setminus \{\varphi\})^4$, Ann. Physics, 147, 198-243, (1983)
- [95] I. M. Gel'fand, M. I. Graev, M. I. Graev and I. I. Pyatetskii-Shapiro, \textit{Representation Theory and Automorphic Functions} (W. B. Saunders Co., Philadelphia-London-Toronto, 1969). · [Zbl 0138.07201](#)
- [96] I.M. Gel'fand, M. I. Graev and N. Ya. Vilenkin, \textit{Generalized Functions}. Vol. 5: \textit{Integral Geometry and Representation Theory} (Academic Press, New York-London, 1966).
- [97] I. M. Gel'fand and N. Ya. Vilenkin, \textit{Generalized Functions}. Vol. 4: \textit{Applications of Harmonic Analysis} (Academic Press, New York-London, 1964).
- [98] Glimm, J.; Jaffe, A., Critical exponents and elementary particles, Comm. Math. Phys., 52, 203-209, (1977)
- [99] J. Glimm and A. Jaffe, \textit{Quantum Physics. A Functional Integral Point of View}, Second edition (Springer, New York, 1987). · [Zbl 0461.46051](#)
- [100] Gori, G.; Trombettoni, A., Conformal invariance in three dimensional percolation, p07014, (2015)
- [101] Gubinelli, M.; Imkeller, P.; Perkowski, N., Paracontrolled distributions and singular PDEs, Forum Math. Pi, 3, e6, (2015) · [Zbl 1333.60149](#)
- [102] Hairer, M., A theory of regularity structures, Invent. Math., 198, 269-504, (2014) · [Zbl 1332.60093](#)
- [103] Hairer, M., Regularity structures and the dynamical Φ^3 model, 1-49, (2016), Somerville, MA
- [104] Hartman, P., On isometries and on a theorem of Liouville, Math. Zeit., 69, 202-210, (1958) · [Zbl 0097.38203](#)
- [105] Hasse, H., Zur Theorie der abstrakten elliptischen Funktionenkörper III. Die Struktur des Meromorphismenrings. Die Riemannsche Vermutung, J. Reine Angew.Math., 175, 193-208, (1936) · [Zbl 0014.24902](#)

- [106] Heemskerk, I.; Polchinski, J., Holographic and Wilsonian renormalization groups, *J. High Energy Phys.*, 6, 031, (2011) · [Zbl 1298.81181](#)
- [107] T. Hida, *\textit{Brownian Motion}*, Applications of Mathematics *\textbf{11}* (Springer-Verlag, New York-Berlin, 1980). · [Zbl 0423.60063](#)
- [108] Hida, T.; Takeyuki; Kubo, I.; Nomoto, H.; Yoshizawa, H., On projective invariance of Brownian motion, *Publ. Res. Inst. Math. Sci. Ser. A*, 4, 595-609, (1969) · [Zbl 0206.19101](#)
- [109] C. Hongler, *\textit{Conformal Invariance of Ising Model Correlations}*, Ph.D. Thesis (University of Geneva, 2010). Available at <http://archive-ouverte.unige.ch/unige:18163/ATTACHMENT01>. · [Zbl 1304.82013](#)
- [110] Hongler, C.; Smirnov, S., The energy density in the planar Ising model, *Acta Math.*, 211, 191-225, (2013) · [Zbl 1287.82007](#)
- [111] Jack, I.; Osborn, H., Constraints on RG flow for four dimensional quantum field theories, *Nucl. Phys. B*, 883, 425-500, (2014) · [Zbl 1323.81071](#)
- [112] Jaffe, A., Stochastic quantization, reflection positivity, and quantum fields, *J. Stat. Phys.*, 161, 1-15, (2015) · [Zbl 1327.81260](#)
- [113] J. Kaplan, *\textit{Lectures on AdS/CFT From The Bottom Up}*, Notes for Fall 2013 and 2015 course at Johns Hopkins University. Available at <http://www.pha.jhu.edu/~jaredk/AdSCFTCourseNotesPublic.pdf>.
- [114] Kennedy, T., Conformal invariance of the 3D self-avoiding walk, *Phys. Rev. Lett.*, 111, 165703, (2013)
- [115] Kos, F.; Poland, D.; Simmons-Duffin, D., Bootstrapping mixed correlators in the 3D Ising model, *J. High Energy Phys.*, 11, 109, (2014) · [Zbl 1392.81202](#)
- [116] A. Kupiainen, *\textit{Introduction to the Renormalization Group}*. April 4, 2014 version of lecture notes available at http://www.math.lmu.de/~bohmmech/Teaching/bricmont2014/notes_kupiainen.pdf.
- [117] Kupiainen, A., Renormalization group and stochastic PDEs, *Ann. Henri Poincaré*, 17, 497-535, (2016) · [Zbl 1347.81063](#)
- [118] Leichtnam, E., Scaling group flow and Lefschetz trace formula for laminated spaces with p-adic transversal, *Bull. Sci. Math.*, 131, 638-669, (2007) · [Zbl 1131.14024](#)
- [119] Lerner, E. Yu., The hierarchical Dyson model and p-adic conformal invariance, *Theor. Math. Phys.*, 97, 1259-1266, (1993) · [Zbl 0840.22038](#)
- [120] Lerner, È Yu. Yu.; Missarov, M. D., p-Adic conformal invariance and the Bruhat-Tits tree, *Lett. Math. Phys.*, 22, 123-129, (1991) · [Zbl 0746.22018](#)
- [121] Letac, G., Seul le groupe des similitudes-inversions prserve le type de la loi de Cauchy-conforme de \mathbb{R}^n pour $n > 1$, *J. Funct. Anal.*, 68, 43-54, (1986) · [Zbl 0612.60019](#)
- [122] Lévy, P., Le mouvement brownien plan, *Amer. J. Math.*, 62, 487-550, (1940) · [Zbl 66.0619.02](#)
- [123] Lodhia, A.; Sheffield, S.; Sun, X.; Watson, S. S., Fractional Gaussian fields: a survey, *Probab. Surv.*, 13, 1-56, (2016) · [Zbl 1334.60055](#)
- [124] Lohmann, M.; Slade, G.; Wallace, B., Critical two-point function for long-range $\mathcal{O}(\mathbb{Z}_n)$ models below the upper critical dimension, *J. Stat. Phys.*, 169, 1132-1161, (2017) · [Zbl 1387.82012](#)
- [125] E. Lukacs, *\textit{Characteristic Functions}*, Second edition, revised and enlarged (Hafner Publishing Co., New York, 1970). · [Zbl 0201.20404](#)
- [126] Lyons, T. J., Differential equations driven by rough signals, *Rev. Mat. Iberoamericana*, 14, 215-310, (1998) · [Zbl 0923.34056](#)
- [127] Magnen, J.; Unterberger, J., From constructive field theory to fractional stochastic calculus. (I) An introduction: rough path theory and perturbative heuristics, *Ann. Henri Poincaré*, 12, 1199-1226, (2011) · [Zbl 1231.81058](#)
- [128] Magnen, J.; Unterberger, J., From constructive field theory to fractional stochastic calculus. (II) Constructive proof of convergence for the Lévy area of fractional Brownian motion with Hurst index $\alpha \in (1/8, 1/4)$, *Ann. Henri Poincaré*, 13, 209-270, (2012) · [Zbl 1264.81272](#)
- [129] P. Major, *\textit{Multiple Wiener-Itô Integrals. With Applications to Limit Theorems}*, *Lect. Notes in Math.* *\textbf{849}* (Springer, Cham, 2014). · [Zbl 1301.60004](#)
- [130] Manin, Yu. I., p-Adic automorphic functions, *J. Soviet Math.*, 5, 279-333, (1976) · [Zbl 0375.14007](#)
- [131] Manin, Yu. I., Three-dimensional hyperbolic geometry as \mathbb{Z} -adic Arakelov geometry, *Invent. Math.*, 104, 223-243, (1991) · [Zbl 0754.14014](#)
- [132] Manin, Yu.; Drinfeld, V., Periods of p-adic Schottky groups, *J. Reine Angew. Math.*, 262/263, 239-247, (1973) · [Zbl 0275.14017](#)
- [133] Manin, Yu. I.; Marcolli, M., Holography principle and arithmetic of algebraic curves, *Adv. Theor. Math. Phys.*, 5, 617-650, (2001) · [Zbl 1042.14006](#)
- [134] Y. Meyer, *\textit{Wavelets, Vibrations and Scalings}*, CRM Monograph Series *\textbf{9}* (American Math. Soc., Providence, RI, 1998).
- [135] McBryan, O. A.; Rosen, J., Existence of the critical point in ϕ^4 field theory, *Comm. Math. Phys.*, 51, 97-105, (1976)
- [136] H. McKean and V. Moll, *\textit{Elliptic Curves. Function Theory, Geometry, Arithmetic}* (Cambridge Univ. Press, Cambridge, 1999). · [Zbl 0967.11002](#)
- [137] Missarov, M. D., Critical indices for models with long-range interaction, *Teor. Mat. Fiz.*, 46, 232-241, (1981)
- [138] Missarov, M. D., The spectrum of the differential of a renormalization group, *Teor. Mat. Fiz.*, 57, 406-413, (1983)
- [139] P. K. Mitter, "Long range ferromagnets: renormalization group analysis," Slides of 10/24/2013 talk at LPTHE, Université Pierre

- [140] Mourrat, J.-C.; Weber, H., Global well-posedness of the dynamic F4 model in the plane, *Ann. Probab.*, 45, 2398-2476, (2017) · [Zbl 1381.60098](#)
- [141] Nakayama, Y., Scale invariance vs conformal invariance, *Phys. Rep.*, 569, 1-93, (2015)
- [142] Neeb, K.-H.; Ólafsson, G., Reflection positivity and conformal symmetry, *J. Funct. Anal.*, 266, 2174-2224, (2014) · [Zbl 1290.22006](#)
- [143] Newman, C. M., Normal fluctuations and the FKG inequalities, *Comm. Math. Phys.*, 74, 119-128, (1980) · [Zbl 0429.60096](#)
- [144] Newman, C. N., A general central limit theorem for FKG systems, *Comm. Math. Phys.*, 91, 75-80, (1983) · [Zbl 0528.60024](#)
- [145] Nyikos, P. J., Metrizability and the Fréchet-Urysohn property in topological groups, *Proc. Amer. Math. Soc.*, 83, 793-801, (1981) · [Zbl 0474.22001](#)
- [146] Osborn, H., Weyl consistency conditions and a local renormalisation group equation for general renormalisable field theories, *Nucl. Phys. B*, 363, 486-526, (1991)
- [147] Osborne, M. S., On the Schwartz-Bruhat space and the Paley-Wiener theorem for locally compact abelian groups, *J. Funct. Anal.*, 19, 40-49, (1975) · [Zbl 0295.43008](#)
- [148] J. Palis, Jr. and W. de Melo, *Geometric Theory of Dynamical Systems. An introduction* (Springer-Verlag, New York-Berlin, 1982).
- [149] Paulos, M. F.; Rychkov, S.; Rees, B. C.; Zan, B., Conformal invariance in the long-range Isingmodel, *Nucl. Phys. B*, 902, 246-291, (2016) · [Zbl 1332.82017](#)
- [150] M. Picco, "Critical behavior of the Ising model with long range interactions," Preprint arXiv:1207.1018[cond-mat.stat-mech], (2012).
- [151] Rajabpour, M. A., Conformal symmetry in non-local field theories, *J. High Energy Phys.*, 6, 076, (2011) · [Zbl 1298.81149](#)
- [152] Rattazzi, R.; Rychkov, V. S.; Tonni, E.; Vichi, A., Bounding scalar operator dimensions in 4D CFT, *J. High Energy Phys.*, 12, 031, (2008) · [Zbl 1329.81324](#)
- [153] J. G. Ratcliffe, *Foundations of Hyperbolic Manifolds*, Graduate Texts in Mathematics **149** (Springer, New York, 2006).
- [154] Read, C. J., Quantum field theories in all dimensions, *Comm. Math. Phys.*, 177, 631-671, (1996) · [Zbl 0851.46051](#)
- [155] Rehren, K.-H., Comments on a recent solution to Wightman's axioms, *Comm. Math. Phys.*, 178, 453-465, (1996) · [Zbl 0892.46085](#)
- [156] D. Ruelle, *Statistical Mechanics: Rigorous Results* (W. A. Benjamin, Inc., New York-Amsterdam, 1969). · [Zbl 0177.57301](#)
- [157] S. Rychkov, "Conformal bootstrap in three dimensions?," Preprint arXiv:1111.2115[hep-th], (2011).
- [158] Sak, J., Recursion relations and fixed points for ferromagnets with long-range interactions, *Phys. Rev. B*, 8, 281-285, (1973)
- [159] Sak, J., Low-temperature renormalization group for ferromagnets with long-range interactions, *Phys. Rev. B*, 15, 4344-4347, (1977)
- [160] Sally, P. J., An introduction to p -adic fields, harmonic analysis and the representation theory of SL_2 , *Lett. Math. Phys.*, 46, 1-47, (1998) · [Zbl 0924.22009](#)
- [161] Schwartz, L., Théorie des distributions et transformation de Fourier, *Ann. Univ. Grenoble. Sect. Sci. Math. Phys. (N.S.)*, 23, 7-24, (1948) · [Zbl 0030.12601](#)
- [162] Schwartz, L., Sur l'impossibilité de la multiplication des distributions, *C. R. Acad. Sci. Paris*, 239, 847-848, (1954) · [Zbl 0056.10602](#)
- [163] L. Schwartz, *Théorie des distributions*, Publications de l'Institut de Mathématique de l'Université de Strasbourg, No. IX-X, nouvelle édition, entièrement corrigée, refondue et augmentée (Hermann, Paris, 1966). · [Zbl 0149.09501](#)
- [164] Shlosman, S. B., The method of reflection positivity in the mathematical theory of first-order phase transitions, *Russian Math. Surv.*, 41, 83-134, (1986)
- [165] Siburg, K. F.; Stoimenov, P. A., A measure of mutual complete dependence, *Metrika*, 71, 239-251, (2010) · [Zbl 1182.62124](#)
- [166] Simmons-Duffin, D., A semidefinite program solver for the conformal bootstrap, *J. High Energy Phys.*, 174, 31, (2015)
- [167] Simon, B., Distributions and their Hermite expansions, *J. Math. Phys.*, 12, 140-148, (1971) · [Zbl 0205.12901](#)
- [168] B. Simon, *Functional Integration and Quantum Physics*, Pure and Applied Mathematics **86** (Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York-London, 1979).
- [169] Slade, G., Critical exponents for long-range $OS(n)$ models below the upper critical dimension, *Comm. Math. Phys.*, 358, 343-436, (2018) · [Zbl 1391.82022](#)
- [170] Slade, G.; Tomberg, A., Critical correlation functions for the 4-dimensional weakly self-avoiding walk and n -component $|q|=4$ model, *Comm. Math. Phys.*, 342, 675-737, (2016) · [Zbl 1342.82070](#)
- [171] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That*, Corrected third printing of the 1978 edition, Princeton Landmarks in Physics (Princeton Univ. Press, Princeton, NJ, 2000). · [Zbl 1026.81027](#)
- [172] T. Tao, "Dyadic models," Blog post of 07/27/2007. Available at <https://terrytao.wordpress.com/2007/07/27/dyadic-models/>.
- [173] Unterberger, J., Mode d'emploi de la théorie constructive des champs bosoniques, avec une application aux chemins rugueux, [How to use constructive bosonic field theory, with an application to rough paths], *Confluentes Math.*, 4, 34, (2012) · [Zbl](#)

- [174] Unterberger, J., A renormalized rough path over fractional Brownian motion, *Comm. Math. Phys.*, 320, 603-636, (2013) · [Zbl 1276.60048](#)
- [175] Valdivia, M., Representaciones de los espacios $D^s(\Omega)$ y $D'(\Omega)$, *Rev. Real Acad. Cienc. Exact. Fís. Natur. Madrid*, 72, 385-414, (1978)
- [176] V. S. Vladimirov, I. V. Volovich and E. I. Zelenov, *p-Adic Analysis and Mathematical Physics*, Series on Soviet and East European Mathematics 1 (World Scientific Publ. Co., Inc., River Edge, NJ, 1994). · [Zbl 0812.46076](#)
- [177] Vogt, D.; Zapata, G. I. (ed.), Sequence space representations of spaces of test functions and distributions, 405-443, (1983), New York
- [178] Waldschmidt, M., Sur les méthodes de Schneider, Gel'fond et Baker, *Séminaire de Théorie des Nombres de Bordeaux*, 16, 1-14, (1988) · [Zbl 0714.11042](#)
- [179] Walsh, J. B., An introduction to stochastic partial differential equations, *École d'été de Probabilités de Saint-Flour*, XIV-1984, 1180, 265-439, (1986)
- [180] Weil, A.; Krieger, M. H., A 1940 Letter of André Weil on analogy in Mathematics, Excerpted from *Doing Mathematics*, 52, 334-341, (2005)
- [181] Wilker, J. B.; Davis, C. (ed.); Grünbaum, B. (ed.); Sherk, F. A. (ed.), *Inversive geometry*, 379-442, (1981), New York-Berlin
- [182] Wilson, K. G.; Kogut, J., The renormalization group and the ϵ expansion, *Phys. Rep.*, 12, 75-199, (1974)
- [183] Witten, E., Anti de Sitter space and holography, *Adv. Theor. Math. Phys.*, 2, 253-291, (1998) · [Zbl 0914.53048](#)
- [184] Zabrodin, A. V., Non-Archimedean strings and Bruhat-Tits trees, *Comm. Math. Phys.*, 123, 463-483, (1989) · [Zbl 0676.22006](#)
- [185] Zamolodchikov, A. B., Conformal symmetry and multicritical points in two-dimensional quantum field theory, *Soviet J. Nucl. Phys.*, 44, 529-533, (1987)

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