

**Ciarlet, P. G.**

**A new class of variational problems arising in the modeling of elastic multi-structures.**  
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The modeling of junctions between elastic structures is an important practical problem. In this paper elastic multistuctures whose substructures have different dimensions (three-dimensional bodies, two-dimensional plates and shells, one-dimensional rods) are analysed as a coupled pluri-dimensional problem. A three-dimensional elastic structure consisting of a three-dimensional portion connected to a thin plate is treated in detailed form. A limit problem is formulated to solve the variational equations of the above mentioned elastic multi-structure. The obtained numerical results are discussed and some computations are also analysed. The main result of the paper is a new variational formulation of multi-structures; by using this formulation the problem is simultaneously posed over a three-dimensional set and a two-dimensional set.

Reviewer: [I.Ecsedi](#)

**MSC:**

- [74S30](#) Other numerical methods in solid mechanics (MSC2010)
- [74P10](#) Optimization of other properties in solid mechanics
- [74E30](#) Composite and mixture properties
- [65K10](#) Numerical optimization and variational techniques
- [65N30](#) Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

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**Keywords:**

substructures of different dimensions; coupled, pluri-dimensional, variational problems; elliptic boundary value problems; junctions; elastic multistuctures; three-dimensional bodies; two-dimensional plates; shells; one-dimensional rods; limit problem

**Full Text:** [DOI](#) [EuDML](#)

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