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**Three-dimensional doubly diffusive convectons: instability and transition to complex dynamics.** (English) [Zbl 1419.76214](#)

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Summary: Three-dimensional doubly diffusive convection in a closed vertically extended container driven by competing horizontal temperature and concentration gradients is studied by a combination of direct numerical simulation and linear stability analysis. No-slip boundary conditions are imposed on all six container walls. The buoyancy number  $N$  is taken to be  $-1$  to ensure the presence of a conduction state. The primary instability is subcritical and generates two families of spatially localized steady states known as convectons. The convectons bifurcate directly from the conduction state and are organized in a pair of primary branches that snake within a well-defined range of Rayleigh numbers as the convectons grow in length. Secondary instabilities generating twist result in secondary snaking branches of twisted convectons. These destabilize the primary convectons and are responsible for the absence of stable steady states, localized or otherwise, in the subcritical regime. Thus all initial conditions in this regime collapse to the conduction state. As a result, once the Rayleigh number for the primary instability of the conduction state is exceeded, the system exhibits an abrupt transition to large-amplitude relaxation oscillations resembling bursts with no hysteresis. These numerical results are confirmed here by determining the stability properties of both convecton types as well as the domain-filling states. The number of unstable modes of both primary and secondary convectons of different lengths follows a pattern that allows the prediction of their stability properties based on their length alone. The instability of the convectons also results in a dramatic change in the dynamics of the system outside the snaking region that arises when the twist instability operates on a time scale faster than the time scale on which new rolls are nucleated. The results obtained are expected to be applicable in various pattern-forming systems exhibiting localized structures, including convection and shear flows.

**MSC:**

[76E06](#) Convection in hydrodynamic stability

[76R10](#) Free convection

[76F06](#) Transition to turbulence

[37N10](#) Dynamical systems in fluid mechanics, oceanography and meteorology

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