

Huang, Jingyin; Osajda, Damian

Metric systolicity and two-dimensional Artin groups. (English) Zbl 07087154

Math. Ann. 374, No. 3-4, 1311-1352 (2019).

Summary: We introduce the notion of metrically systolic simplicial complexes. We study geometric and large-scale properties of such complexes and of groups acting on them geometrically. We show that all two-dimensional Artin groups act geometrically on metrically systolic complexes. As direct corollaries we obtain new results on two-dimensional Artin groups and all their finitely presented subgroups: we prove that the Conjugacy Problem is solvable, and that the Dehn function is quadratic. We also show several large-scale features of finitely presented subgroups of two-dimensional Artin groups, lying background for further studies concerning their quasi-isometric rigidity.

MSC:

- 20F65 Geometric group theory
- 20F36 Braid groups; Artin groups
- 20F67 Hyperbolic groups and nonpositively curved groups
- 20F06 Cancellation theory of groups; application of van Kampen diagrams
- 20F10 Word problems, other decision problems, connections with logic and automata (group-theoretic aspects)

Full Text: [DOI](#) [arXiv](#)

References:

- [1] Appel, K.I.: On Artin groups and Coxeter groups of large type. In: Contributions to Group Theory, Contemporary Mathematics, vol. 33, pp. 50-78. American Mathematical Society, Providence, RI (1984) · [Zbl 0576.20021](#)
- [2] Appel, KI; Schupp, PE, Artin groups and infinite Coxeter groups, *Invent. Math.*, 72, 201-220, (1983) · [Zbl 0536.20019](#)
- [3] Brady, N., Crisp, J.: Two-dimensional Artin groups with CAT(0) dimension three. In: Proceedings of the Conference on Geometric and Combinatorial Group Theory, Part I (Haifa, 2000), pp. 185-214 (2002) · [Zbl 1070.20043](#)
- [4] Bell, RW, Three-dimensional FC Artin groups are CAT(0), *Geom. Dedic.*, 113, 21-53, (2005) · [Zbl 1134.20038](#)
- [5] Bestvina, M., Non-positively curved aspects of Artin groups of finite type, *Geom. Topol.*, 3, 269-302, (1999) · [Zbl 0998.20034](#)
- [6] Bridson, M.R., Haefliger, A.: Metric Spaces of Non-positive Curvature, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 319. Springer, Berlin (1999)
- [7] Brady, T.; McCammond, JP, Three-generator Artin groups of large type are biautomatic, *J. Pure Appl. Algebra*, 151, 1-9, (2000) · [Zbl 1004.20023](#)
- [8] Brady, T.; McCammond, JP, Braids, posets and orthoschemes, *Algebraic Geom. Topol.*, 10, 2277-2314, (2010) · [Zbl 1205.05246](#)
- [9] Brady, T., Artin groups of finite type with three generators, *Mich. Math. J.*, 47, 313-324, (2000) · [Zbl 0996.20022](#)
- [10] Brieskorn, E.; Saito, K., Artin-gruppen und Coxeter-gruppen, *Invent. Math.*, 17, 245-271, (1972) · [Zbl 0243.20037](#)
- [11] Charney, R.; Crisp, J., Automorphism groups of some affine and finite type Artin groups, *Math. Res. Lett.*, 12, 321-333, (2005) · [Zbl 1077.20055](#)
- [12] Charney, R., Davis, M.W.: Finite $K(\langle \pi \rangle; 1)$ s for Artin groups. In: Prospects in Topology (Princeton, NJ, 1995), pp. 110-124 (1994) · [Zbl 0930.55006](#)
- [13] Charney, R.; Davis, MW, The $K(\langle \pi \rangle; 1)$ -problem for hyperplane complements associated to infinite reflection groups, *J. Am. Math. Soc.*, 8, 597-627, (1995) · [Zbl 0833.51006](#)
- [14] Crisp, J.; Godelle, E.; Wiest, B., The conjugacy problem in subgroups of right-angled Artin groups, *J. Topol.*, 2, 442-460, (2009) · [Zbl 1181.20030](#)
- [15] Charney, R., Artin groups of finite type are biautomatic, *Math. Ann.*, 292, 671-683, (1992) · [Zbl 0736.57001](#)
- [16] Charney, R., Geodesic automation and growth functions for Artin groups of finite type, *Math. Ann.*, 301, 307-324, (1995) · [Zbl 0813.20042](#)
- [17] Chepoi, V., Graphs of some CAT(0) complexes, *Adv. Appl. Math.*, 24, 125-179, (2000) · [Zbl 1019.57001](#)
- [18] Chermak, A., Locally non-spherical Artin groups, *J. Algebra*, 200, 56-98, (1998) · [Zbl 0901.20025](#)
- [19] Chepoi, V.; Osajda, D., Dismantlability of weakly systolic complexes and applications, *Trans. Am. Math. Soc.*, 367, 1247-1272, (2015) · [Zbl 1376.20047](#)

- [20] Conner, GR, Discreteness properties of translation numbers in solvable groups, *J. Group Theory*, 3, 77-94, (2000) · [Zbl 0956.20039](#)
- [21] Charney, R.; Paris, L., Convexity of parabolic subgroups in Artin groups, *Bull. Lond. Math. Soc.*, 46, 1248-1255, (2014) · [Zbl 1308.20037](#)
- [22] Crisp, J., Automorphisms and abstract commensurators of 2-dimensional Artin groups, *Geom. Topol.*, 9, 1381-1441, (2005) · [Zbl 1135.20027](#)
- [23] Deligne, P., Les immeubles des groupes de tresses généralisées, *Invent. Math.*, 17, 273-302, (1972) · [Zbl 0238.20034](#)
- [24] Digne, F., Présentations duales des groupes de tresses de type affine \tilde{A} , *Comment. Math. Helv.*, 81, 23-47, (2006) · [Zbl 1143.20020](#)
- [25] Digne, F., A Garside presentation for Artin-Tits groups of type C_n , *Ann. Inst. Fourier (Grenoble)*, 62, 641-666, (2012) · [Zbl 1260.20056](#)
- [26] Elsner, T., Flats and the at torus theorem in systolic spaces, *Geom. Topol.*, 13, 661-698, (2009) · [Zbl 1228.20033](#)
- [27] Garside, FA, The braid group and other groups, *Q. J. Math. Oxf. Ser. (2)*, 20, 235-254, (1969) · [Zbl 0194.03303](#)
- [28] Gersten, SM; Short, H., Small cancellation theory and automatic groups. II, *Invent. Math.*, 105, 641-662, (1991) · [Zbl 0734.20014](#)
- [29] Haglund, F.: Complexes simpliciaux hyperboliques de grande dimension. Prepublication Orsay, vol. 71 (2003)
- [30] Haettel, T.; Kielak, D.; Schwer, P., The 6-strand braid group is CAT(0), *Geom. Dedic.*, 182, 263-286, (2016) · [Zbl 1347.20044](#)
- [31] Hanlon, RG; Martínez-Pedroza, E., Lifting group actions, equivariant towers and subgroups of non-positively curved groups, *Algebraic Geom. Topol.*, 14, 2783-2808, (2014) · [Zbl 1335.20045](#)
- [32] Hermiller, S.; Meier, J., Algorithms and geometry for graph products of groups, *J. Algebra*, 171, 230-257, (1995) · [Zbl 0831.20032](#)
- [33] Huang, J., Osajda, D.: Large-type Artin groups are systolic, preprint arXiv:1706.05473 (2017)
- [34] Huang, J., Osajda, D.: Quasi-Euclidean tilings over 2-dimensional Artin groups and their applications, preprint arXiv:1711.00122 (2017)
- [35] Januszkiewicz, T.; Świątkowski, J., Simplicial nonpositive curvature, *Publ. Math. Inst. Hautes Études Sci.*, 104, 1-85, (2006) · [Zbl 1143.53039](#)
- [36] Januszkiewicz, T.; Świątkowski, J., Filling invariants of systolic complexes and groups, *Geom. Topol.*, 11, 727-758, (2007) · [Zbl 1188.20043](#)
- [37] Lyndon, R.C., Schupp, P.E.: Combinatorial group theory. In: *Classics in Mathematics*. Springer, Berlin (2001). Reprint of the 1977 edition · [Zbl 0997.20037](#)
- [38] McCammond, J., Dual euclidean Artin groups and the failure of the lattice property, *J. Algebra*, 437, 308-343, (2015) · [Zbl 1343.20039](#)
- [39] McCammond, J.; Sulway, R., Artin groups of Euclidean type, *Invent. Math.*, 210, 231-282, (2017) · [Zbl 1423.20032](#)
- [40] Osajda, D.; Przytycki, P., Boundaries of systolic groups, *Geom. Topol.*, 13, 2807-2880, (2009) · [Zbl 1271.20056](#)
- [41] Osajda, D.; Przytuła, T., Classifying spaces for families of sub-groups for systolic groups, *Groups Geom. Dyn.*, 12, 1005-1060, (2018) · [Zbl 06941810](#)
- [42] Peifer, D., Artin groups of extra-large type are biautomatic, *J. Pure Appl. Algebra*, 110, 15-56, (1996) · [Zbl 0872.20036](#)
- [43] Pride, SJ, On Tits' conjecture and other questions concerning Artin and generalized Artin groups, *Invent. Math.*, 86, 347-356, (1986) · [Zbl 0633.20021](#)
- [44] Petrunin, A., Stadler, S.: Metric minimizing surfaces revisited, preprint arXiv:1707.09635 (2017)
- [45] Servatius, H., Automorphisms of graph groups, *J. Algebra*, 126, 34-60, (1989) · [Zbl 0682.20022](#)
- [46] Świątkowski, J., Regular path systems and (bi)automatic groups, *Geom. Dedic.*, 118, 23-48, (2006) · [Zbl 1165.20036](#)
- [47] VanWyk, L., Graph groups are biautomatic, *J. Pure Appl. Algebra*, 94, 341-352, (1994) · [Zbl 0812.20018](#)
- [48] van der Lek, H.: *The Homotopy Type of Complex Hyperplane Complements*. Katholieke Universiteit te Nijmegen, Nijmegen (1983)
- [49] Wise, D.T.: *Sixtolic complexes and their fundamental groups (2003)* \textbf{(unpublished manuscript)}
- [50] Zeeman, EC, Relative simplicial approximation, *Proc. Camb. Philos. Soc.*, 60, 39-43, (1964) · [Zbl 0119.38502](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.